Annotated SAS Output (ASO)

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Abstract

The analyses for 15 data sets are done using the statistical software package SAS. The focus of the annotated SAS output (ASO) is upon specifying appropriate input statements to obtain desired output. The 15 analyses include problems involving multiple linear regression, polynomial regression with lack-of-fit, comparison of regression lines and analysis of covariance. The analysis of treatment means is demonstrated for one-way classifications and two-way factorial treatment designs from completely randomized and randomized complete block experimental designs having equal and unequal replication. Treatment means are also analyzed from split-plot, repeated measures and crossover experiments.

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Introduction

The present form of the Annotated SAS Output (ASO) has evolved from the original project aimed at illustrating common statistical methods using the statistical software package SAS (BU-664-M, BU-705-M, and BU-814-M). The primary goal of these annotated outputs has been to provide students with direction in using a statistical package to analyze data at the level of Statistics 602. This expanded version of the ASO should be of use to students as well as others outside the classroom.

Over the past six years there have been many people who have contributed to the ASO. Most of these people have been either students, lab instructors or undergraduate assistants involved with the conduct of Statistics 602. This list includes Anna Angelos, Suzanne Aref, Valerie Arneson, Jim Babb, Calvin Berry, Margaret Cecce, Patricia Firey, Laura Gnazzo, Walter Kremers, George Legall, Jon Maatta, Charles McCulloch, Patricia Nolan, Norma Phalen, Walter Piegorsch, Beth Snelbaker, and David Umbach.

Description

A total of 15 data sets are employed to illustrate the application of SAS in the analysis of data. The data sets are derived from actual designed or observational experiments.

The name of the data set, which appears in the Table of Contents, and a description of the type of analysis that it serves to illustrate are provided below. The first 10 data sets are taken from Analyzing Experimental Data by Regression (Allen and Cady, 1982), while references are given for the sources of the remaining five.

1) Arsenic Data illustrate straight-line regression for the mean-slope and intercept-slope models.
2) Firefly Data illustrate multiple linear regression with two predictor variables. Model sequences are demonstrated and partial leverage and residual plots are shown.
3) Soymilk Data illustrate polynomial regression and lack-of-fit from a completely randomized design. Orthogonal polynomial contrast coefficients are calculated for unequally spaced treatment levels. These contrasts are compared with the sequential model sums of squares.

4) Electricity Load Data illustrate a model sequence when several straight lines may need to be fitted.

5) Potato Leafhopper Data illustrate an analysis of treatment means via a complete set of orthogonal contrasts. The experiment design is a one-way classification in a completely randomized design with equal replication.

6) Lymphocyte Data illustrate an analysis of a 2 x 2 factorial experiment laid out in a completely randomized design with equal replication.

7) Fat Digestibility Data illustrate an analysis of a 2 x 2 factorial experiment laid out in a randomized complete block design.

8) Protein Nutrition Data illustrate an analysis of a one-way completely randomized design with unequal replication. A set of non-orthogonal contrasts are presented as well as a complete orthogonal set.

9) Swamp pH Data illustrate the analysis of cell means in a 2 x 3 factorial experiment with unequal replication. The first analysis follows that presented in Allen and Cady (1982). Several ANOVA tables are considered as are the weighted and unweighted cell means (the MEANS and LSMEANS of SAS). The second approach demonstrates an analysis of unweighted means which is discussed in Snedecor and Cochran (1980).

10) Soybean Physiological Data illustrate a covariate analysis. Several ways are shown to estimate treatment means adjusted and unadjusted for the covariate. The "classical" ANCOVA table is given as is a test for homogeneity of slopes.

11) Potato Scab Data illustrate an analysis of a 2 x 3 factorial experiment where one treatment factor is qualitative with two levels and the other treatment factor is quantitative with three unequally spaced levels. Two equivalent analyses are presented: the first is a model sequence approach comparing quadratic regression curves (see Allen and Cady, Unit 19); the second is an analysis of cell means wherein appropriate single degree-of-freedom orthogonal polynomial contrasts and interaction contrasts are estimated. These data are taken from a larger set presented in Cochran and Cox (1957, p.97).

12) Alcohol-Drug Data illustrate the analysis of a split-unit experiment. The cell means model is fitted as described in Allen and Cady (1982, p.280). The approach presented allows the simple

effects to be estimated more easily than in a default model specification. The whole-unit analysis proceeds by analyzing the sums and the split-unit analysis is accomplished after removing the whole-unit variability. Such an approach can result in substantial savings of computing dollars and a strengthening of the understanding of such experiments. The usual approach is presented for comparison.

13) Urea Synthesis Data illustrate the analysis of a repeated measures experiment with two treatment groups of unequal numbers, and two repeated measurements upon each experimental unit. The pertinent hypothesis tests may be reduced to three two-sample t-tests based upon the within subject sums or differences. Such an approach tends to render the analysis more understandable than does the usual ANOVA. The ANOVA table is also presented for completeness. These data are taken from Brogan and Kutner (1980).

14) Hemoglobin Data illustrate the analysis of a two-period crossover experiment with unequal numbers of experimental units in the two groups. The calculations for this design are exactly the same as those of the preceding Urea Synthesis Data. The distinction between the two designs is in the method of treatment allocation. These data are taken from Grizzle (1965).

15) Milk Yield Data illustrate the analysis of a three-period three treatment crossover design which is balanced for first order carryover effects. The layout is in repeated Latin squares. Treatment means adjusted and unadjusted for carryover effects are given as are their standard errors. These data are taken from Cochran and Cox (1957, p.135).

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* See description for references.

ARSENIC DATA — Units 4, 5 and 6; Annotated Computer Output, p. 346 (BMDP)

```
TITLE ARSENIC DATA;
DATA ARSENIC;
   INPUT DIST ARSENIC;
   DEV = DIST - 16.1;
   XO = 1;
CARDS;
 2 3.19
 4 3.26
 8 1.82
10 1.02
12 1.85
15 2.05
21 1.34
23 0.79
30 0.66
36 0.30
```

Prints a heading on each page.

These statements create a data set called ARSENIC which contains four variables:
ARSENIC is the observed arsenic level,
DIST is the distance from the arsenic source,
DEV is the difference between Dist and the mean distance (16.1),
XO is a one for each observation.

```
(A) PROC PRINT;
      VAR XO DEV DIST ARSENIC;
```

PROC PRINT prints the variables XO, DEV, DIST, and ARSENIC.

ARSENIC DATA

(A)

OBS	XO	DEV	DIST	ARSENIC
1	1	-14.1	2	3.19
2	1	-12.1	4	3.26
3	1	-8.1	8	1.82
4	1	-6.1	10	1.02
5	1	-4.1	12	1.85
6	1	-1.1	15	2.05
7	1	4.9	21	1.34
8	1	6.9	23	0.79
9	1	13.9	30	0.66
10	1	19.9	36	0.30

(B) These two columns are used in model (B) to predict arsenic levels.

(C) These two columns are used in model (C).

(D) This column is used alone in model (D).

(E) This column is used alone in model (E).

```
    PROC REG DATA=ARSENIC;
(B)   MODEL ARSENIC = XO DEV/NOINT SEQB P CLM SS1 SS2;
         OUTPUT OUT=NEW1 PREDICTED=YHAT1 RESIDUAL=RESID1
            L95M=LOWER  U95M=UPPER STDP=SEPRED STUDENT=STDRES;
(C)   MODEL ARSENIC = DIST/SEQB P SS1 SS2;
(D)   MODEL ARSENIC = DIST/NOINT SEQB P SS1 SS2;
(E)   MODEL ARSENIC = XO/NOINT ;
         OUTPUT OUT=NEW2 PREDICTED=YBAR;
```

PROC REG is a regression procedure; four models are illustrated B - E:
(B) is a mean slope model.
(C) is an intercept slope model.
(D) is regression through the origin.
(E) is a mean model.

The OUTPUT statements create new data sets called NEW1 and NEW2. Each new data set contains all variables in the original data set plus the variables named in the output statement.

```
    PROC PLOT DATA = NEW1;
(G)   PLOT RESID1*YHAT/VREF=0;
(H)   PLOT ARSENIC*DIST='*'   YHAT*DIST='P'
           UPPER*DIST='U'     LOWER*DIST='L'/OVERLAY;
```

Notice that we have directed SAS to use the data set NEW1 for these plots. If we do not specify a data set for a procedure, SAS will use the most recently created data set, in this case SAS would have used NEW2.
Plot (G) is a residual plot for models (B) and (C).
VREF = 0 draws a horizontal line at RESID = 0.
Plot (H) prints observed, predicted, and confidence limits on the same set of axes using the symbols *, P, U and L.

```
    DATA DECOMP;
      MERGE NEW1 NEW2;
       RES1 = YHAT1 - YBAR;

    PROC PRINT DATA=DECOMP;
(I)   VAR ARSENIC YBAR RES1 RESID1 STDRES SEPRED;
        TITLE DATA DECOMPOSITION FOR ARSENIC DATA ANALYSIS;
```

DECOMP is a new data set which is a combination, or MERGE of NEW1 and NEW2. DECOMP contains all the variables in NEW1 and NEW2 and the new variable.
RES1 is the difference between the mean arsenic level and the predicted arsenic level from model (B) or (C).

PROC PRINT prints the data decomposition.

MEAN AND [illegible] MODEL

(B) MODEL ARSENIC = XO DEV/NOINT P SEQB SS1 SS2 CIM;

SEQUENTIAL PARAMETER ESTIMATES ← The option SEQB produces the sequential b's.

```
XO      1.628           ← Coefficient for the mean model (see p.35).
DEV     1.628 -.078151  ← Coefficients for the mean and slope model
                          (see pp. 39 and 45).
```

Note that using DEV instead of DIST as the X variable produces sequential b's that are the same as the partials. See (C).

DEP VARIABLE: ARSENIC

ANOVA:

SOURCE	DF	SUM OF SQUARES	MEAN SQUARE	F VALUE	PROB>F
MODEL	2	33.386414	16.693207	57.061	0.0001
ERROR	8	2.340386	0.292548 = s^2		
U TOTAL	10	35.726800			

The option NOINT instructs SAS not to supply an intercept since XO has been included in the model. When XO is in the model and the NOINT option is used, the model SS includes the SS for the mean.

— U TOTAL stands for UNCORRECTED TOTAL. ROOT MSE is the standard deviation.

```
ROOT MSE    0.540877 = s      R-SQUARE    0.9345
DEP MEAN    1.628000 = ȳ      ADJ R-SQ    0.9263  ← when NOINT is used, the reported R² is incorrect.
C.V.        33.22342
(Coefficient of variation)
NOTE: NO INTERCEPT TERM IS USED. R-SQUARE IS REDEFINED.
```

VARIABLE	DF	PARTIAL PARAMETER ESTIMATE	STANDARD ERROR	T FOR H0: PARAMETER=0	PROB > |T|	TYPE I SS
XO	1	1.628000 = $\bar{y}$	0.171040	9.518	0.0001	26.503840 = R(0)
DEV	1	-0.078151 = slope	0.016112	-4.850	0.0013	6.882574 = R(DEV|0)

The options SS1 and SS2 in the MODEL statement produce the TYPE I SS (the sequential SS) and the TYPE II SS (the partial SS).

VARIABLE	DF	TYPE II SS
XO	1	26.503840 = R(0|DEV)
DEV	1	6.882574 = R(DEV|0)

The P option in the MODEL statement prints each observed value, predicted value, and residual.

OBS	ACTUAL	PREDICT VALUE	STD ERR PREDICT	LOWER95% MEAN	UPPER95% MEAN	RESIDUAL
1	3.190	2.730	0.284371	2.074	3.386	0.460075
2	3.260	2.574	0.259352	1.976	3.172	0.686377
3	1.820	2.261	0.215145	1.765	2.757	-.441020
4	1.020	2.105	0.197265	1.650	2.560	-1.085
5	1.850	1.948	0.183354	1.526	2.371	-.098418
6	2.050	1.714	0.171956	1.317	2.111	0.336034
7	1.340	1.245	0.188382	0.810647	1.679	0.094938
8	0.790000	1.039	0.203997	0.618339	1.559	-.238760
9	0.660000	0.541706	0.281803	-.108140	1.192	0.118294
10	0.300000	0.072801	0.363402	-.765212	0.910815	0.227199

```
SUM OF RESIDUALS            7.07767E-15
SUM OF SQUARED RESIDUALS    2.340386
```

Should be exactly zero.

The CIM option prints the upper and lower confidence limits for a 95% confidence interval on μ at each observed X.

When NOINT is used, the reported R^2 is incorrect. $R^2 = \frac{\text{Corrected Model SS}}{\text{Corrected Total SS}}$ where both model and total SS have been corrected for the mean. NOINT causes the SS for the mean to be included in both numerator and denominator. To calculate the correct R^2, subtract the SS for the mean ($n\bar{y}^2$) from the MODEL SS and TOTAL SS.

Ex.: Model (C) $n\bar{y}^2 = 26.503840$

$$R^2 = \frac{33.386414 - n\bar{y}^2}{35.726800 - n\bar{y}^2} = 0.7462$$

(compare to above R^2)

$$R^2 = \frac{\text{Corrected Model SS}}{\text{Corrected Total SS}} = 1 - \frac{\text{RESIDUAL SS}}{\text{Corrected Total SS}}$$

$$R^2_{adj} = 1 - \left[\frac{(\text{Resid SS}) * \left(\frac{n-1}{n-p}\right)}{\text{Corrected Total SS}}\right]$$

where n is the number of observations and p is the number of variables fitted, including the intercept or XO.

INTERCEPT [illegible] SLOPE MODEL

Ⓒ MODEL ARSENIC = DIST/P SEQB SS1 SS2;

SEQUENTIAL PARAMETER ESTIMATES

INTERCEP	1.628	
DIST	2.88623	-.078151

} same sequentials as Ⓑ

[The NOINT option is not used, so SAS supplies an intercept. The model SS does not include the SS for the mean ($n\bar{y}^2$).]

DEP VARIABLE: ARSENIC

SOURCE	DF	SUM OF SQUARES	MEAN SQUARE	F VALUE	PROB>F
MODEL	1	6.682574	6.682574	23.526	0.0004
ERROR	8	2.340366	0.292548 = s^2, same as Ⓑ		
C TOTAL	9	9.022960			

→Corrected total SS

ROOT MSE	s = 0.540877	R-SQUARE	0.7462 ← Correct R^2.
DEP MEAN	$\bar{y}$ = 1.628000	ADJ R-SQ	0.7145 ← The adjusted R^2 is "adjusted" for the number of variables in the model.
C.V.	33.22342		

VARIABLE	DF	PARTIAL PARAMETER ESTIMATE	STANDARD ERROR	T FOR H0: PARAMETER=0	PROB > |T|	(SEQUENTIAL) TYPE I SS
INTERCEP	1	2.886226	0.310720	9.289	0.0001	26.503840
DIST	1	-0.078151	0.016112	-4.850	0.0013	6.682574

↳same as Ⓑ

VARIABLE	DF	(PARTIAL) TYPE II SS
INTERCEP	1	25.241773
DIST	1	6.682574

↳same as Ⓑ

OBS	ACTUAL	PREDICT VALUE	RESIDUAL
1	3.190	2.730	0.460075
2	3.260	2.574	0.686377
3	1.820	2.261	-.441030
4	1.020	2.105	-1.085
5	1.850	1.948	-.098418
6	2.050	1.714	0.336034
7	1.340	1.245	0.094936
8	0.790000	1.089	-.298760
9	0.660000	0.541706	0.118294
10	0.300000	0.072801	0.227199

same as Ⓑ

SUM OF RESIDUALS 8.40994E-15

SUM OF SQUARED RESIDUALS 2.340366

The parameter estimates are different from those in Ⓑ, but the TYPE I SS are the same.

PROC REG does not compute F tests for the TYPE I or TYPE II SS. The t tests of the parameter estimates are equivalent to F tests of the TYPE II SS. To perform the independent F tests of the TYPE I SS, form the ratio

$$\frac{(\text{TYPE I SS})/\text{df}}{(\text{Residual SS})/(\text{Residual df})} = F^{\text{df}}_{\text{Resid df}}$$

LINE THR[illegible] ORIGIN

SEQUENTIAL PARAMETER ESTIMATES

DIST .0467975 = slope of line through origin (see p. 53)

DEP VARIABLE: ARSENIC

includes SS for the mean

SOURCE	DF	SUM OF SQUARES	MEAN SQUARE	F VALUE	PROB>F
MODEL	1	8.144641	8.144641	2.658	0.1375
ERROR	9	27.582159	3.064684 = s^2, compare to previous models		
U TOTAL	10	35.726800			

ROOT MSE	1.750624	R-SQUARE	~~0.2280~~
DEP MEAN	1.628000	ADJ R-SQ	~~0.2280~~
C.V.	107.5322		

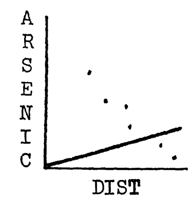
NOTE: NO INTERCEPT TERM IS USED. R-SQUARE IS REDEFINED.

VARIABLE	DF	PARAMETER ESTIMATE	STANDARD ERROR	T FOR H0: PARAMETER=0	PROB > |T|	(SEQUENTIAL) TYPE I SS
DIST	1	0.046798	0.028706	1.630	0.1375	8.144641

VARIABLE	DF	(PARTIAL) TYPE II SS
DIST	1	8.144641

Ⓓ ARSENIC = DIST/NOINT;

NOINT is used, so SAS does not supply the intercept, and XO is not specified in the model. Result: fitting a straight line through the origin.



Such a model does not make much sense with the ARSENIC data set, although regression through the origin is useful in other circumstances.

MEAN MODEL

Ⓔ ARSENIC = XO/NOINT

DEP VARIABLE: ARSENIC

SS for the mean, equal to $n\bar{y}^2 = 10 * (1.628)^2$

SOURCE	DF	SUM OF SQUARES	MEAN SQUARE	F VALUE	PROB>F
MODEL	1	26.503840	26.503840	25.863	0.0007
ERROR	9	9.222960	1.024773		
U TOTAL	10	35.726800			

Note that s^2 for this model is larger than for models Ⓑ and Ⓒ. Fitting the slope reduces the estimate of σ^2.

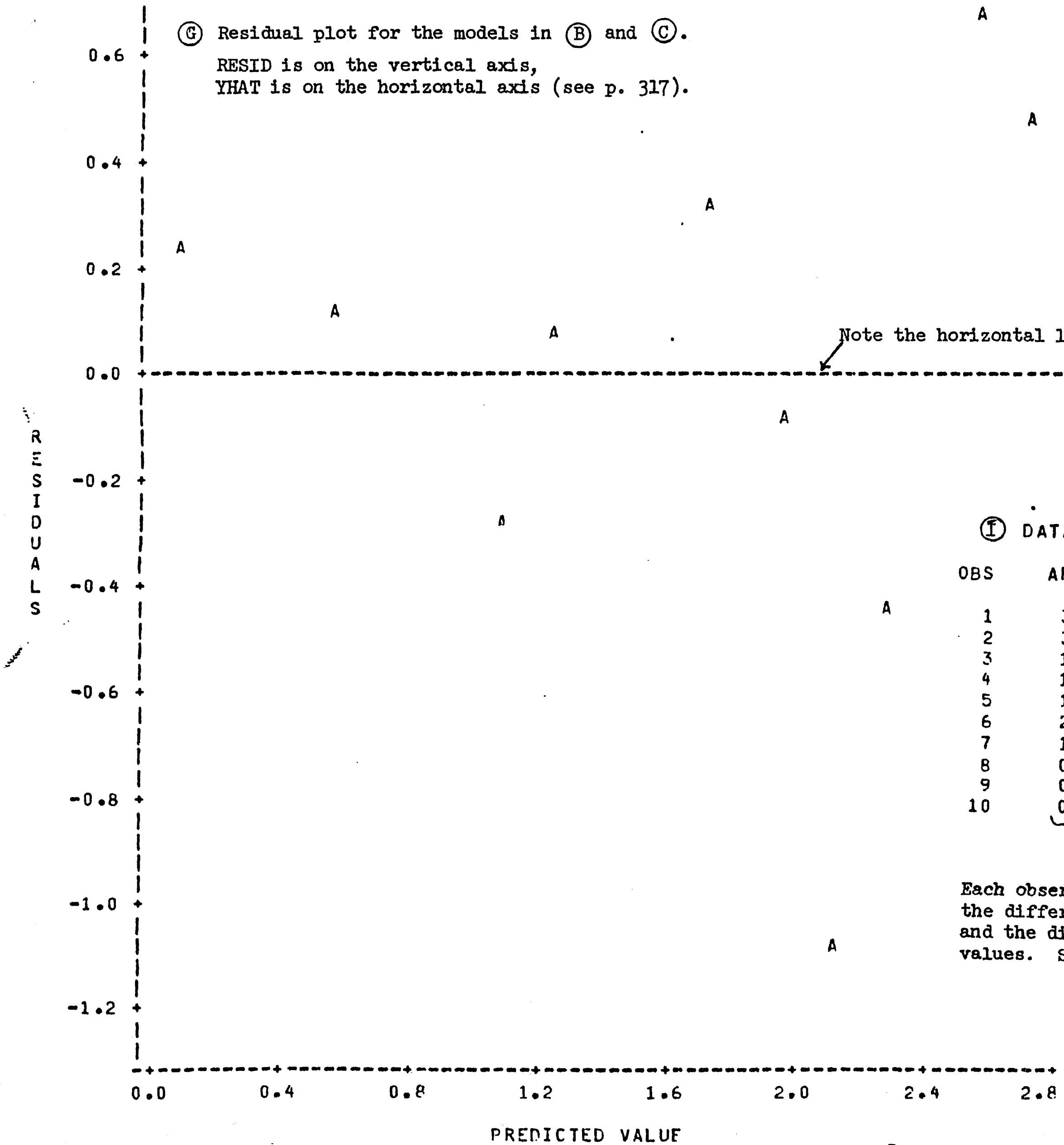
ROOT MSE	1.012311	R-SQUARE	0.7418
DEP MEAN	1.628000	ADJ R-SQ	0.7418
C.V.	62.18126		

NOTE: NO INTERCEPT TERM IS USED. R-SQUARE IS REDEFINED.

VARIABLE	DF	PARAMETER ESTIMATE	STANDARD ERROR	T FOR H0: PARAMETER=0	PROB > |T|	TYPE I SS
X0	1	1.628000	0.320121	5.086	0.0007	0

This model was run so that the predicted values, each equal to $\bar{y} = 1.628$, could be saved in the data set ONE, to be used in the data decomposition, part Ⓘ.

PLOT OF RESID1*YHAT1 LEGEND: A = 1 OBS, B = 2 OBS, ETC.



Ⓖ Residual plot for the models in Ⓑ and Ⓒ.
RESID is on the vertical axis,
YHAT is on the horizontal axis (see p. 317).

Note the horizontal line drawn by VREF = 0

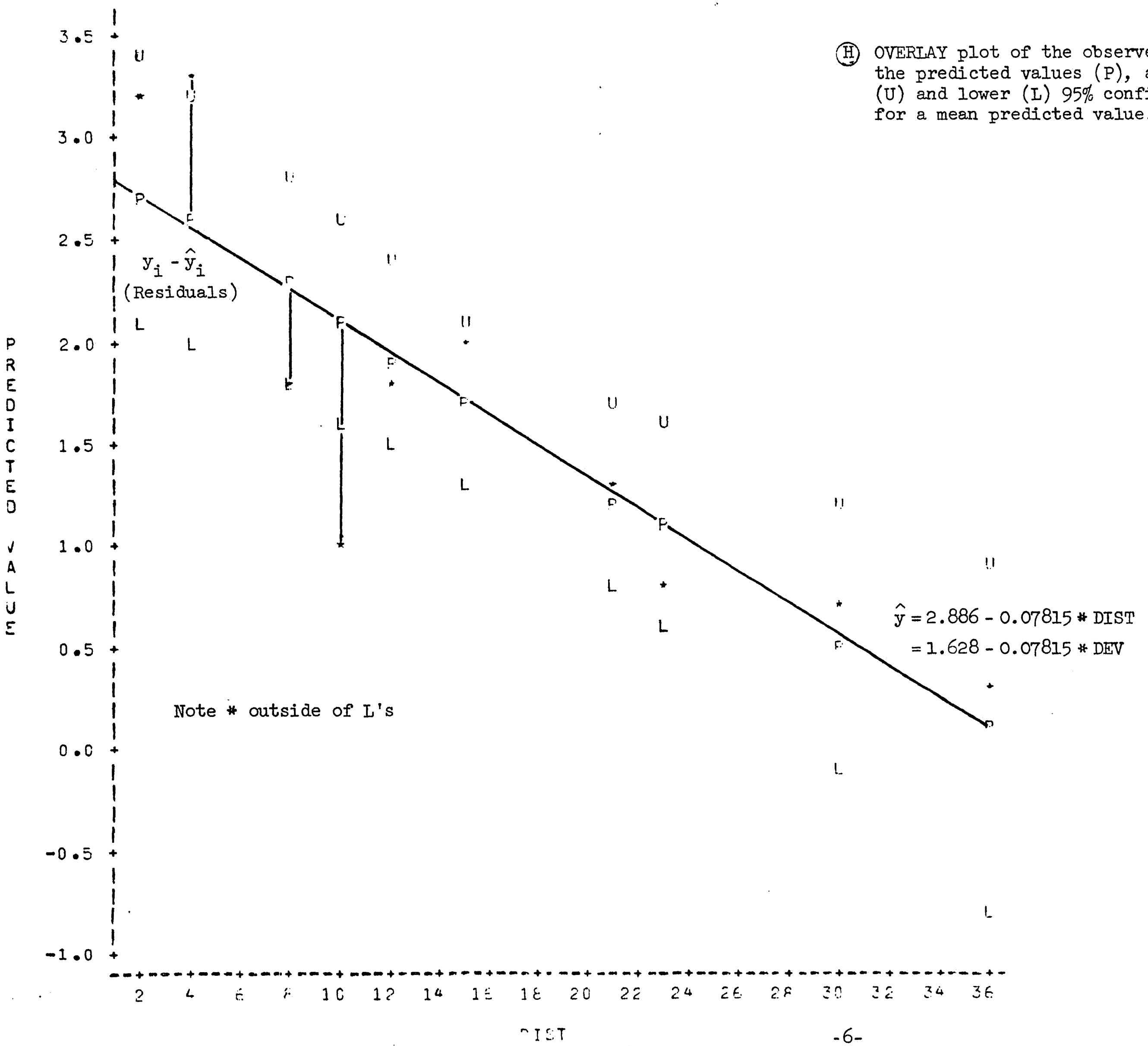
Ⓘ DATA DECOMPOSITION FOR ARSENIC DATA ANALYSIS

OBS	ARSENIC	YBAR	RES1	RESID1	STDRES	SEPRED
1	3.19	1.628	1.1019	0.4601	1.0000	0.284371
2	3.26	1.628	0.9456	0.6864	1.4461	0.259352
3	1.82	1.628	0.6330	-0.4410	-0.8887	0.215145
4	1.02	1.628	0.4767	-1.0847	-2.1538	0.197268
5	1.85	1.628	0.3204	-0.0984	-0.1934	0.183354
6	2.05	1.628	0.0860	0.3360	0.6553	0.171956
7	1.34	1.628	-0.3829	0.0949	0.1873	0.188382
8	0.79	1.628	-0.5392	-0.2988	-0.5964	0.203997
9	0.66	1.628	-1.0863	0.1183	0.2562	0.281803
10	0.30	1.628	-1.5552	0.2272	0.5671	0.363402

$$y_1 = \bar{y} + (\hat{y}_1 - \bar{y}) + (y_1 - \hat{y}_1)$$

Each observation can be decomposed into the overall mean, the difference between the predicted values and the mean, and the difference between the observed and predicted values. See Unit 6, p. 50.

PLOT OF ARSENIC*DIST SYMBOL USED IS *
PLOT OF YHAT1*DIST SYMBOL USED IS P
PLOT OF UPPER*DIST SYMBOL USED IS U
PLOT OF LOWER*DIST SYMBOL USED IS L



(H) OVERLAY plot of the observed values (*), the predicted values (P), and the upper (U) and lower (L) 95% confidence limits for a mean predicted value.

FIREFLY DATA – Units 10, 12, and 13; ACO, p. 351 (SAS)

```
TITLE FIREFLY DATA;
DATA FIREFLY;
  INPUT FTIME LIGHT TEMP @@;
  XO=1;
  CARDS;
45 26 21.1   40 35 23.9   58 40 17.8   50 41 22.0   31 45 22.3
52 55 23.3   38 56 25.5   54 55 20.5   40 70 21.7   28 75 26.7
38 79 25.0   36 87 24.4   36 100 22.3 46 100 25.5 40 110 26.7
31 130 25.5 40 140 26.7
```

Creates the data set FIREFLY.
FTIME is the response variable; LIGHT and TEMP are explanatory variables.
We used @@ to tell SAS to read the entire line because we have more than one observation on each line.

```
 PROC PRINT;
A  VAR XO LIGHT TEMP FTIME;
```

— Prints the X : Y matrix for C.

```
 PROC PLOT;
B  PLOT FTIME*LIGHT FTIME*TEMP LIGHT*TEMP;
```

— 3 separate plots, FTIME against each explanatory variable and LIGHT vs. TEMP.

```
 PROC REG;
C  MODEL FTIME=XO LIGHT TEMP/NOINT P SS1 SS2 SEQB;
         OUTPUT OUT=NEW1 PREDICTED=YHAT1 RESIDUAL=RESID1;
D  MODEL FTIME=TEMP LIGHT/P CLM SS1 SS2 SEQB PARTIAL TOL;
E  MODEL FTIME=TEMP/P SS1 SS2 SEQB;
         OUTPUT OUT=NEW2 PREDICTED=YHAT2 RESIDUAL=RESID2;
```

PROC REG used to fit models with both LIGHT and TEMP, and then a reduced model.

```
DATA NEWALL;   MERGE NEW1 NEW2;
PROC PLOT DATA=NEWALL;
F PLOT RESID1*YHAT1/VREF=0;
G PLOT RESID2*TEMP/VREF=0;
```

F — Residual plot for C and D.
G — Residual plot for E.

FIREFLY DATA

OBS	XO	LIGHT	TEMP	FTIME
1	1	26	21.1	45
2	1	35	23.9	40
3	1	40	17.8	58
4	1	41	22.0	50
5	1	45	22.3	31
6	1	55	23.3	52
7	1	56	25.5	38
8	1	55	20.5	54
9	1	70	21.7	40
10	1	75	26.7	28
11	1	79	25.0	38
12	1	87	24.4	36
13	1	100	22.3	36
14	1	100	25.5	46
15	1	110	26.7	40
16	1	130	25.5	31
17	1	140	26.7	40

A $\underset{\sim}{X} : \underset{\sim}{Y}$ matrix for the general model:
FTIME = XO LIGHT TEMP/NOINT in part C.

FULL MODEL

(C) FTIME = XO LIGHT TEMP/NOINT

FIREFLY DATA

SEQUENTIAL PARAMETER ESTIMATES ← The sequential coefficients are the diagonal elements of the SEQB output (see p. 99).

XO	41.3529 = $\bar{y}$		← See p. 94.
LIGHT	48.8962	-.103083	← Intercept and slope coefficients for FTIME = XO LIGHT (see p. 95).
TEMP	91.4745	6.9E-04 -2.12753	← Intercept and two slopes which define a tilted plane (see p. 99).

Note that the slope in the LIGHT direction changes sign when TEMP is added to the model.

DEP VARIABLE: FTIME

SOURCE	DF	SUM OF SQUARES	MEAN SQUARE	F VALUE	PROB>F
MODEL	3	29527.857	9842.619	201.710	0.0001
ERROR	14	683.143	48.795924 = s^2		
U TOTAL	17	30211.000 ← Includes SS for the mean.			

ROOT MSE	6.985408	R-SQUARE	~~0.9774~~
DEP MEAN	41.352941	ADJ R-SQ	~~0.9742~~
C.V.	16.89217		

NOTE: NO INTERCEPT TERM IS USED. R-SQUARE IS REDEFINED.

VARIABLE	DF	PARAMETER ESTIMATE	STANDARD ERROR	T FOR H0: PARAMETER=0	PROB > |T|
XO (→ Intercept)	1	91.474451	18.825103	4.859	0.0003
LIGHT	1	0.0006919891	0.068339	0.010	0.9921
TEMP	1	-2.127529	0.917599	-2.319	0.0361

→ Partial coefficients.

Slope of plane in LIGHT direction, see p. 100.
Slope of plane in TEMP direction, see p. 100.

VARIABLE	DF	TYPE I SS	TYPE II SS
XO	1	29071.118 = R(0)	1152.148 = R(0|T,L)
LIGHT	1	194.421 = R(L|0)	0.005003143 = R(L|T,0)
TEMP	1	262.318 = R(T|L,0)	262.318 = R(T|L,0)

When LIGHT is fitted last, it does not account for much of the remaining SS.

OBS	ACTUAL	PREDICT VALUE	STD ERR PREDICT	LOWER95% MEAN	UPPER95% MEAN	RESIDUAL
1	45.000	46.602	2.972	40.227	52.976	-1.602
2	40.000	40.651	3.280	33.616	47.685	-.650720
3	58.000	53.632	4.517	43.944	63.321	4.368
4	50.000	44.697	2.373	39.607	49.787	5.303
5	31.000	44.062	2.235	39.268	48.855	-13.062
6	52.000	41.941	2.015	37.620	46.262	10.059
7	38.000	37.261	3.170	30.462	44.061	0.738795
8	54.000	47.898	2.795	41.904	53.892	6.102
9	40.000	45.356	2.326	40.367	50.344	-5.356
10	28.000	34.721	3.256	27.737	41.705	-6.721
11	38.000	38.341	2.011	34.028	42.653	-.340885
12	36.000	39.623	1.843	35.670	43.576	-3.623
13	36.000	44.100	3.231	37.170	51.030	-8.100
14	46.000	37.292	2.259	32.447	42.136	8.708
15	40.000	34.746	2.821	28.695	40.796	5.254
16	31.000	37.312	3.478	29.853	44.772	-6.312
17	40.000	34.766	3.847	26.516	43.016	5.234

SUM OF RESIDUALS 3.94351E-13
SUM OF SQUARED RESIDUALS 683.1429

FULL MODEL

D FTIME = TEMP LIGHT

SEQUENTIAL PARAMETER ESTIMATES

INTERCEP	$\boxed{41.3529}$ = $\bar{y}$		
TEMP	91.3816	$\boxed{-2.12144}$ ← Intercept and slope coefficients for FTIME = TEMP (no NOINT).	
LIGHT	91.4745	-2.12753	$\boxed{6.9E-04}$ ← same as C.

DEP VARIABLE: FTIME

Note that the sequential coefficients from ABDO are on the diagonal of the SEQB output. See p. 100.

SOURCE	DF	SUM OF SQUARES	MEAN SQUARE	F VALUE	PROB>F
MODEL	2	456.739	228.370	4.680	0.0278
ERROR	14	683.143	48.795924 = s^2, same as C.		
C TOTAL	16	1139.882			

same ANOVA as C, except that the MODEL and TOTAL SS are corrected for the mean.

ROOT MSE	6.985408	R-SQUARE	$\boxed{0.4007}$
DEP MEAN	41.352941	ADJ R-SQ	$\boxed{0.3151}$
C.V.	16.89217		

VARIABLE	DF	PARAMETER ESTIMATE	STANDARD ERROR	T FOR HO: PARAMETER=0	PROB > |T|
INTERCEP	1	91.474451	18.825103	4.859	0.0003
TEMP	1	-2.127529	0.917599	-2.319	0.0361
LIGHT	1	0.0006919891	0.068339	0.010	0.9921

VARIABLE	DF	TYPE I SS	TYPE II SS	TOLERANCE
INTERCEP	1	29071.118	1152.148	.
TEMP	1	456.734	262.318	0.571054
LIGHT	1	0.005003143	0.005003143	0.571054

TYPE I SS:
= R(0)
= R(T|0)
= R(L|T,0)

Again, LIGHT fitted after TEMP accounts for little of the remaining SS.

TYPE II SS:
= R(0|T,L)
= R(T|L,0)
= R(L|T,0)

$$\text{Tol} = \frac{\Sigma\left(X_i - \hat{X}_i(0, \cdots)\right)^2}{\Sigma(X_i - \bar{X})^2} = 1 - R^2_{X_i \cdot \text{all other X's}}$$

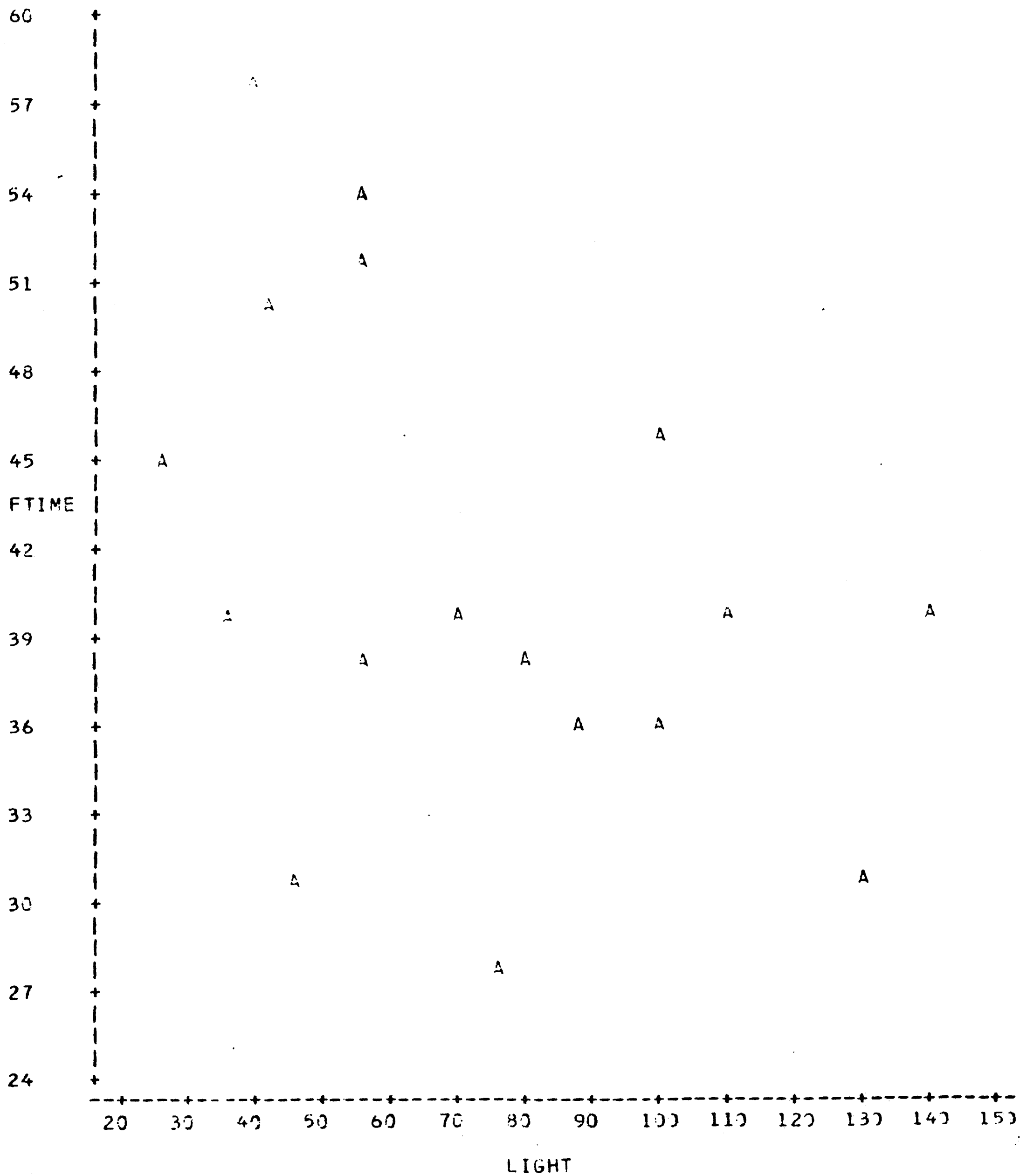
all other X's

Same as C.

FIREFLY DATA

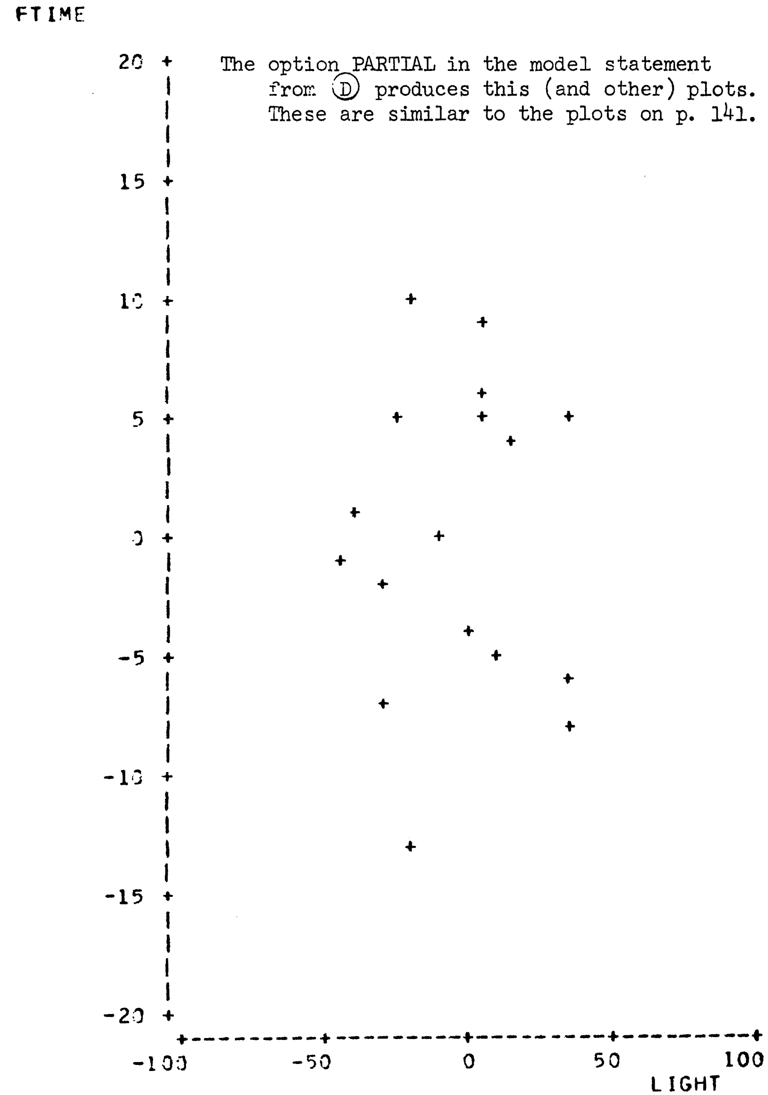
E

PLOT OF FTIME*LIGHT   LEGEND: A = 1 OBS, B = 2 OBS, ETC.



D

PARTIAL REGRESSION RESIDUAL PLOTS



LIGHT - $\widehat{\text{LIGHT}}$ (INTERCEPT, TEMP)

There is little linear trend in this plot, so fitting LIGHT after an intercept and TEMP will account for a small portion of the remaining sum of squares, as in C and D.

REDUCED MODEL

(E) FTIME = XO TEMP/NOINT

SEQUENTIAL PARAMETER ESTIMATES

X0	41.3529 = $\bar{y}$	
TEMP	91.3816	-2.12144 ← same estimates as second line, (D)

DEP VARIABLE: FTIME

SOURCE	DF	SUM OF SQUARES	MEAN SQUARE	F VALUE	PROB>F
MODEL	2	29527.852	14763.926	324.174	0.0001
ERROR	15	683.148	45.543196 = s^2		
U TOTAL	17	30211.000			

Note that s^2 has not increased very much from the s^2 in (C) and (D) where both TEMP and LIGHT were fitted.

ROOT MSE	6.748570	R-SQUARE	0.9774
DEP MEAN	41.352941	ADJ R-SQ	0.9759
C.V.	16.31944		

NOTE: NO INTERCEPT TERM IS USED. R-SQUARE IS REDEFINED.

VARIABLE	DF	PARAMETER ESTIMATE	STANDARD ERROR	T FOR H0: PARAMETER=0	PROB > |T|	TYPE I SS
X0	1	91.381582	15.882436	5.754	0.0001	29071.118
TEMP	1	-2.121444	0.669902	-3.167	0.0064	456.734

Compare with standard errors of the estimates, part (D)

SS for reduced model (FTIME = XO/NOINT) = SS for the mean ($n\bar{y}^2$) ≐ 456.736

VARIABLE	DF	TYPE II SS	TOLERANCE
X0	1	1507.671	1805.465876
TEMP	1	456.734	1.000000

F test for the need of the general model with TEMP and LIGHT over the reduced model with TEMP (pp. 138-140):

$$F'_{14} = \frac{[(\text{Model SS, general model}) - (\text{Model SS, reduced model})]/\text{difference in df between models}}{(\text{Residual SS, general model})/(\text{Residual df, general model})}$$

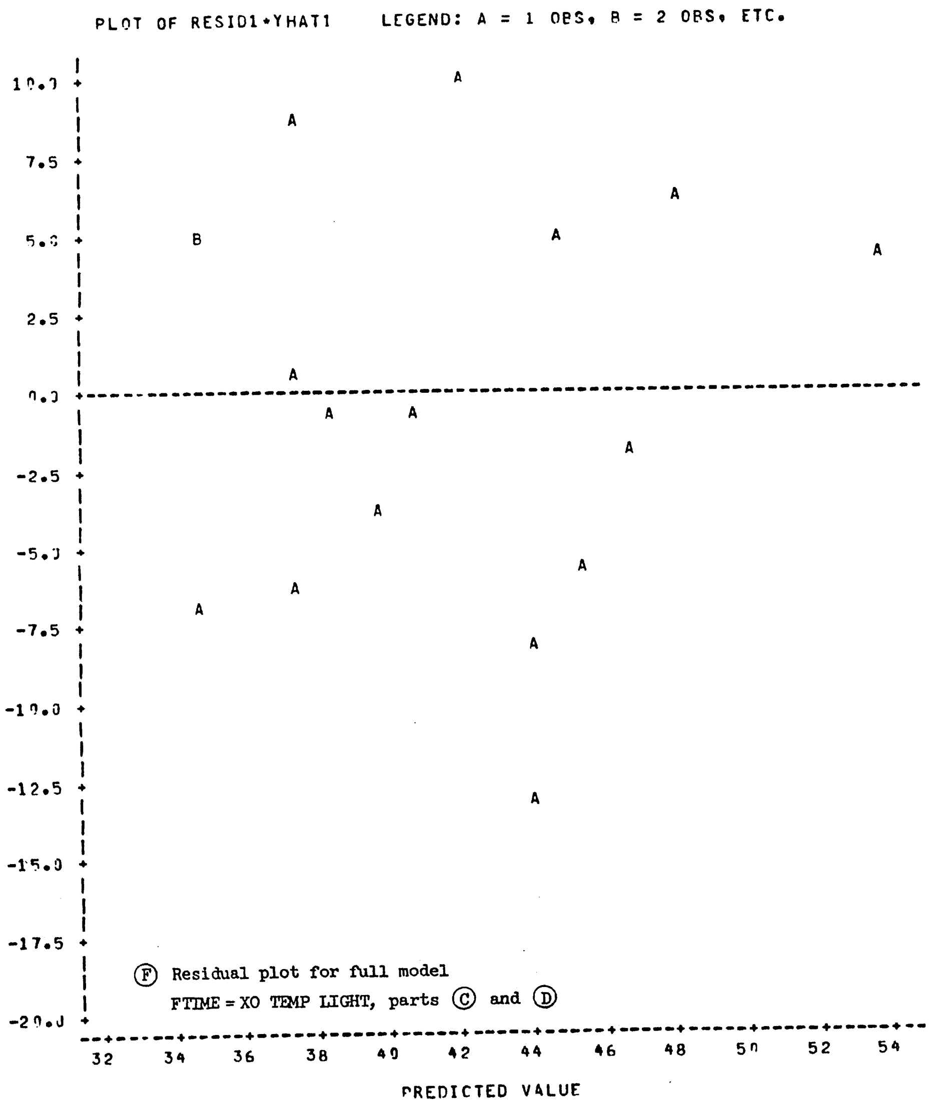
$$= \frac{(456.739 - 456.736)/1}{(683.143)/14} \doteq 0.0001$$ Reduced model is adequate .

This test is equivalent to the t-test of the LIGHT parameter estimate in (C) or (D).

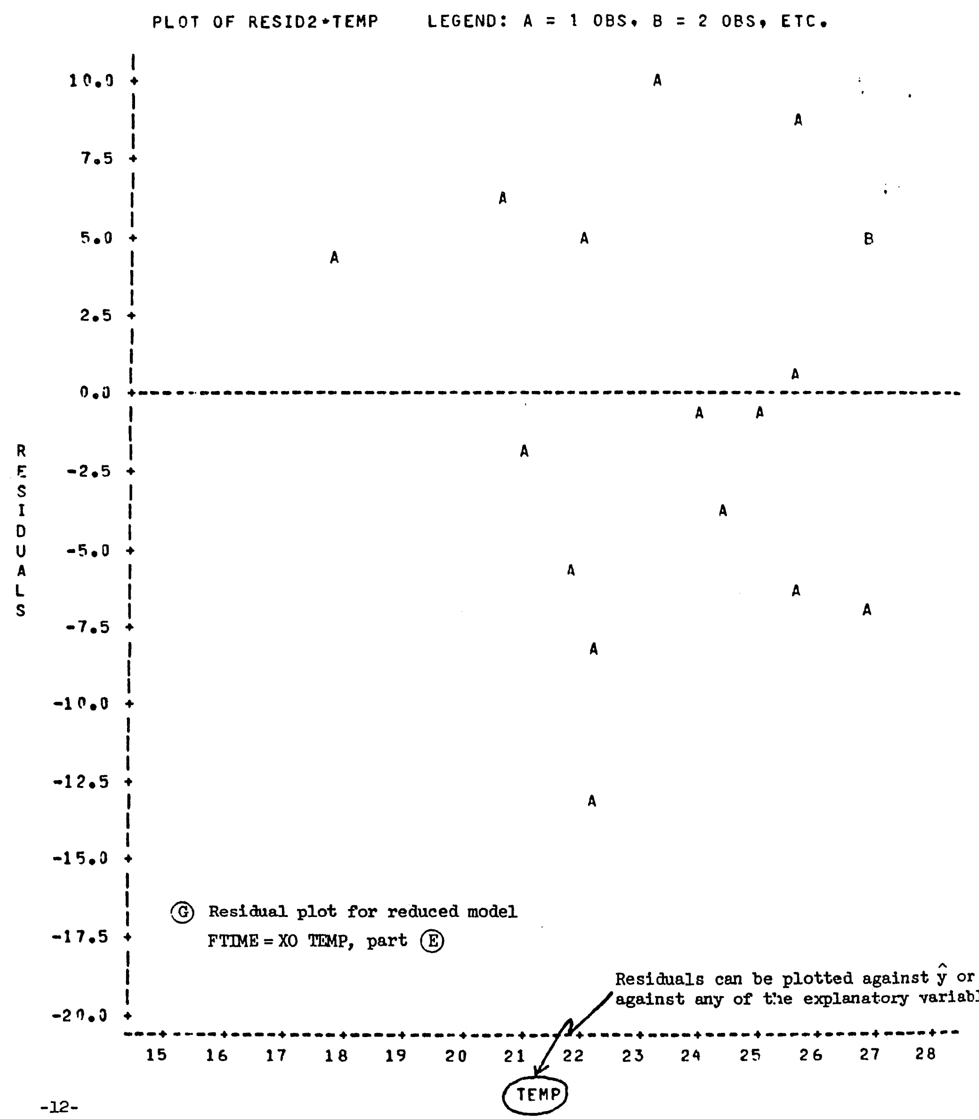
OBS	ACTUAL	PREDICT VALUE	RESIDUAL
1	45.000	46.619	-1.619
2	49.000	49.679	-.679071
3	58.000	53.620	4.380
4	50.000	44.710	5.290
5	31.000	44.073	-13.073
6	52.000	41.952	10.048
7	38.000	37.285	0.715240
8	54.000	47.892	6.108
9	40.000	45.346	-5.346
10	28.000	34.739	-6.739
11	38.000	38.345	-.345482
12	36.000	39.618	-3.618
13	36.000	44.073	-8.073
14	46.000	37.285	8.715
15	40.000	34.739	5.261
16	31.000	37.285	-6.285
17	40.000	34.739	5.261

SUM OF RESIDUALS 3.65930E-13

SUM OF SQUARED RESIDUALS 683.1475



Ⓕ Residual plot for full model
FTIME = XO TEMP LIGHT, parts Ⓒ and Ⓓ



Ⓖ Residual plot for reduced model
FTIME = XO TEMP, part Ⓔ

Residuals can be plotted against $\hat{y}$ or against any of the explanatory variabl

SOYMILK DATA -- Unit 11

```
DATA SOYMILK;
INPUT TIME Y @@;
TIME2 = TIME*TIME;
LOF = TIME; CARDS;

0 2.74 0 2.25 0 2.34 12 3.14 12 2.68 12 2.83
30 3.44 30 3.53 30 3.63 60 3.68 60 3.75 60 3.51
```

In this output the time is coded in minutes. Thus, the coefficients differ from those reported in Unit 11 by a factor of 60 or 60^2 for x and x^2, respectively.

A

```
PROC REG;
     MODEL  Y = TIME TIME2 / SS1 SS2 SEQB;
     OUTPUT OUT=QUAD PREDICTED=YHAT RESIDUAL=RESQ;
```

Fitting a quadratic polynomial as a reduced model.

The sequential and partial SS and regression coefficients are requested.

B

```
PROC GLM; CLASS TIME;
     MODEL  Y = TIME;
     ESTIMATE 'LINEAR' TIME -25.5 -13.5 4.5 34.5 / DIVISOR=2043;
     ESTIMATE 'QUADRATIC' TIME 415.507 -182.379 -539.207 306.079 /
              DIVISOR=590336.7098;
     CONTRAST 'LINEAR' TIME -25.5 -13.5 4.5 34.5;
     CONTRAST 'QUADRATIC' TIME 415.507 -182.379 -539.207 306.079;
     LSMEANS TIME / STDERR;
     OUTPUT OUT=FULL PREDICTED=YBAR RESIDUAL=RESID;
```

Fitting the full (cell means) model.

The ESTIMATE statements compute the sequential coefficients using the orthogonal polynomial contrast coefficients obtained via the ORTHO algorithm (see X and L below).

The CONTRAST statements give the SS associated with the above ESTIMATE statements. Compare with the PROC REG Type I SS.

C, D

```
PROC PLOT;
     PLOT RESID*YBAR=TIME / VREF=0;
     PLOT Y*TIME='*' YHAT*TIME='P' YBAR*TIME='M' / OVERLAY;
```

The residual plot for the full (cell means) model and an overlay of the observed data, the fitted means and the predicted response from the quadratic model.

E

```
PROC GLM; CLASS LOF;
     MODEL  Y = TIME TIME2 LOF;
```

This provides an easy way to assess the Lack-of-Fit due to fitting a lower order polynomial rather than the full (cell means) model.

$$X = \begin{array}{c} \begin{array}{ccc} x_o & x & x^2 \end{array} \\ \begin{bmatrix} 1 & 0 & 0 \\ 1 & 12 & 144 \\ 1 & 30 & 900 \\ 1 & 60 & 3600 \end{bmatrix} \end{array} \xrightarrow[\text{ORTHO}]{} L = \begin{array}{c} \begin{array}{ccc} x_o & x-\bar{x} & x^2-\hat{x}^2(0,x) \end{array} \\ \begin{bmatrix} 1 & -25.5 & 415.507 \\ 1 & -13.5 & -182.379 \\ 1 & 4.5 & -539.207 \\ 1 & 34.5 & 306.079 \end{bmatrix} \end{array}$$

$$\Sigma(x-\hat{x}(0))^2 = 2043$$

$$\Sigma(x^2-\hat{x}^2(0,x))^2 = 590336.7098$$

Note: The computations required in the ORTHO algorithm may be carried out by any regression program. For example, the last column of L may be obtained as the residuals from a regression of x^2 on x and x_o (see p. 109).

These results were found using the REGR command in MINITAB. Compare with L on p. 114 where time is in hours.

(A) PROC REG output for the fitted quadratic polynomial model

SEQUENTIAL PARAMETER ESTIMATES

INTERCEP	3.12667 = $\bar{y}$		
TIME	2.62141	0.019814 = $b_{1\cdot 0}$	
TIME2	2.41049	0.051197	-5.1E-04 = $b_{2\cdot 01}$
	= $b_{0\cdot 12}$	= $b_{1\cdot 02}$	

DEP VARIABLE: Y

SOURCE	DF	SUM OF SQUARES	MEAN SQUARE	F VALUE	PROB>F
MODEL	2	2.862563	1.431282	40.520	0.0001
ERROR	9	0.317903	0.035323		
C TOTAL	11	3.180467			
ROOT MSE		0.187943	R-SQUARE	0.9000	
DEP MEAN		3.126667	ADJ R-SQ	0.8778	
C.V.		6.010972			

VARIABLE	Partial DF	PARAMETER ESTIMATE	STANDARD ERROR	T FOR H0: PARAMETER=0	PROB > |T|
INTERCEP	1	2.410490	0.100670	23.944	0.0001
TIME	1	0.051197	0.009055173	5.654	0.0003
TIME2	1	-0.000507621	0.0001412264	-3.594	0.0058

VARIABLE	DF	TYPE II SS		TYPE I SS	
INTERCEP	1	20.251745	= $R(x_o \mid x, x^2)$	117.313	= $R(x_o)$
TIME	1	1.129142	= $R(x \mid x_o, x^2)$	2.406212	= $R(x \mid x_o)$
TIME2	1	0.456351	= $R(x^2 \mid x_o, x)$	0.456351	= $R(x^2 \mid x_o, x)$

(C) Test for Lack-of-Fit

GENERAL LINEAR MODELS PROCEDURE

CLASS LEVEL INFORMATION

CLASS	LEVELS	VALUES
LOF	4	0 12 30 60

NUMBER OF OBSERVATIONS IN DATA SET = 12

DEPENDENT VARIABLE: Y

SOURCE	DF	SUM OF SQUARES	MEAN SQUARE	F VALUE
MODEL	3	2.88580000	0.96193333	26.12
ERROR	8	0.29466667	0.03683333	PR > F
CORRECTED TOTAL	11	3.18046667		0.0002

R-SQUARE	C.V.	ROOT MSE	Y MEAN
0.907351	6.1382	0.19192012	3.12666667

SOURCE	DF	TYPE I SS	F VALUE	PR > F
TIME	1	2.40621204	65.33	0.0001
TIME2	1	0.45635131	12.39	0.0078
LOF	1	0.02323665	0.63	0.4500

SOURCE	DF	TYPE III SS	F VALUE	PR > F
TIME	0	0.00000000	.	.
TIME2	0	0.00000000	.	.
LOF	1	0.02323665	0.63	0.4500

Note: The F statistic reported above could have been computed from the combined results of parts A and B as

$$F = \frac{[SS(\text{full model}) - SS(\text{reduced model})] / (df_{full} - df_{reduced})}{SS(\text{due to experimental error}) / df_{expt'l\ error}}$$

$$\text{or,}\quad 0.63 = \frac{(2.88580 - 2.862563)/(3 - 2)}{0.29466667/8}$$

GENERAL LINEAR MODELS PROCEDURE

CLASS LEVEL INFORMATION

CLASS	LEVELS	VALUES
TIME	4	0 12 30 60

NUMBER OF OBSERVATIONS IN DATA SET = 12

DEPENDENT VARIABLE: Y

SOURCE	DF	SUM OF SQUARES	MEAN SQUARE	F VALUE
MODEL	3	2.88580000	0.96193333	26.12
ERROR	8	0.29466667	0.03683333 = s^2	PR > F
CORRECTED TOTAL	11	3.18046667		0.0002

Note that s^2 is the pooled estimate of experimental error with 4(3-1) df.

R-SQUARE	C.V.	ROOT MSE	Y MEAN
0.907351	6.1382	0.19192012	3.12666667

SOURCE	DF	TYPE I SS	F VALUE	PR > F
TIME	3	2.88580000	26.12	0.0002

SOURCE	DF	TYPE III SS	F VALUE	PR > F
TIME	3	2.88580000	26.12	0.0002

This is not of any real interest since it is a test of H_o: $\mu_o = \ldots = \mu_{60}$.

CONTRAST	DF	SS	F VALUE	PR > F
LINEAR	1	$R(x\|x_o)$ = 2.40621204	65.33	0.0001
QUADRATIC	1	$R(x^2\|x_o,x)$ = 0.45635231	12.39	0.0078

These SS correspond to the Type I SS reported in PROC REG.

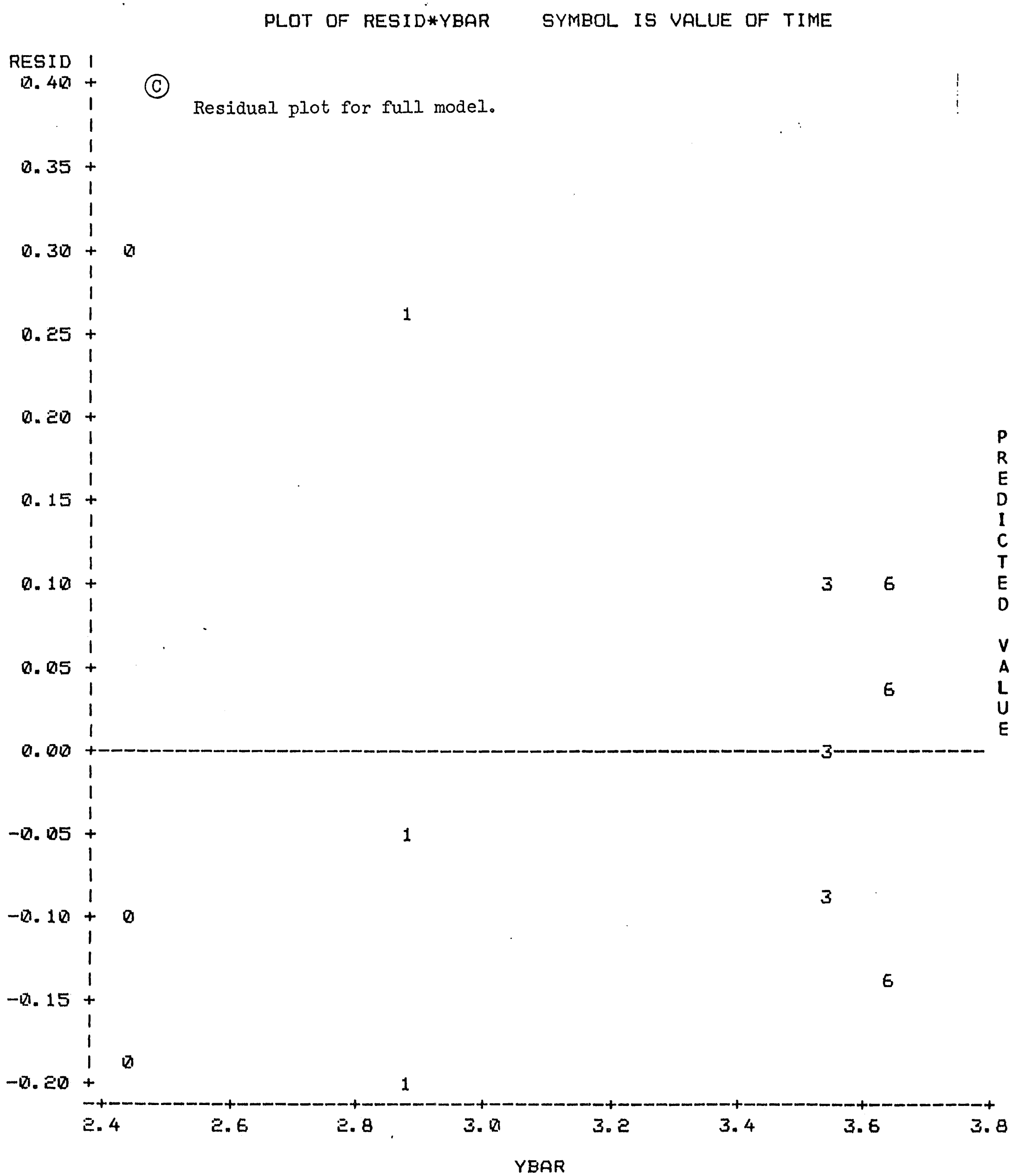
PARAMETER	ESTIMATE	T FOR H0: PARAMETER=0	PR > |T|	STD ERROR OF ESTIMATE
LINEAR	$b_{1.0}$ = 0.01981400	8.08	0.0001	0.00245147
QUADRATIC	$b_{2.01}$ = -0.00050762	-3.52	0.0078	0.00014421

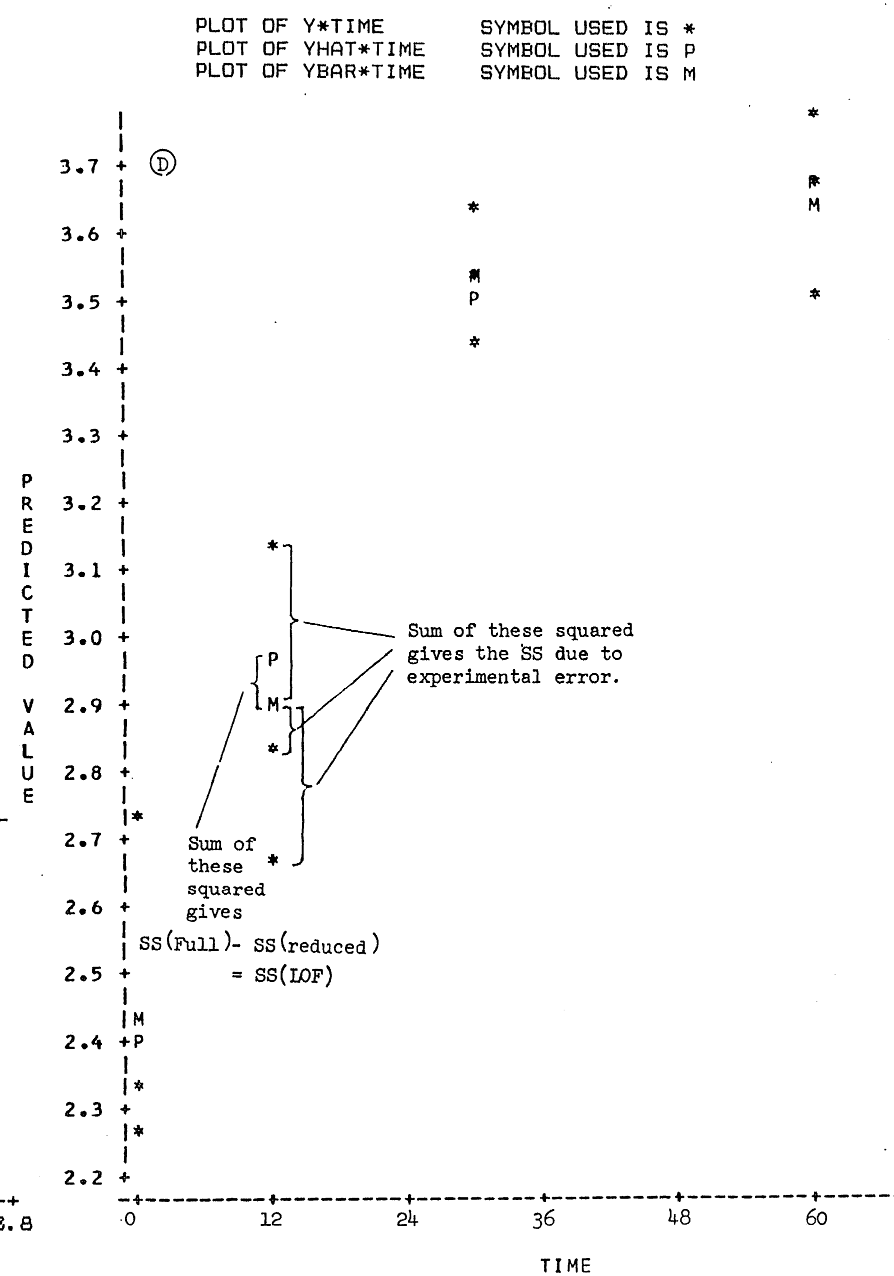
These estimates are the same as the sequential estimates reported in PROC REG via the SEQB option. The standard errors differ because the experimental error is used here, whereas the residual error was used in PROC REG.

LEAST SQUARES MEANS

TIME	Y LSMEAN	STD ERR LSMEAN	~~PROB > |T| H0:LSMEAN=0~~
0	2.44333333	0.11080513	~~0.0001~~
12	2.88333333	0.11080513	~~0.0001~~
30	3.53333333	0.11080513	~~0.0001~~
60	3.64666667	0.11080513	~~0.0001~~

These are the means for each time and their standard errors. Note that SEM = $\sqrt{0.036833/3}$ = 0.1108


PLOT OF RESID*YBAR SYMBOL IS VALUE OF TIME
RESID
(C)
Residual plot for full model.
YBAR


PLOT OF Y*TIME SYMBOL USED IS *
PLOT OF YHAT*TIME SYMBOL USED IS P
PLOT OF YBAR*TIME SYMBOL USED IS M
(D)
PREDICTED VALUE
Sum of these squared gives the SS due to experimental error.
Sum of these squared gives
SS(Full)- SS(reduced) = SS(LOF)
TIME

```
TITLE ELECTRICITY LOAD DATA;
DATA ELECTIC;
 INPUT DATE DAY $ TEMP Y;
 X3=0; X4=0; X5=0; X6=0;   X1=1; X2=TEMP;
 IF DAY='SU' OR DATE=5 THEN DO;
                              X5=1;  X6=TEMP;  X1=0;  X2=0;  END;
 IF DAY ='SA' THEN DO;
                     X3=1;  X4=TEMP; X1=0; X2=0; DAY='Q'; END;
 CARDS;
```

$ indicates that DAY is a character variable.

All statements from a DO to an END are executed as a group.

'SA' is changed to 'Q' for the plot in G.

See A and B for the X variables created by the above statements.

A
```
PROC PRINT;
     VAR X1 X2 X3 X4 X5 X6;
```
Prints the X matrix for the full model: 3 intercepts and 3 slopes.

B
```
PROC PRINT;
     VAR X1 X3 X5 TEMP;
```
Prints the X matrix for the reduced model: 3 intercepts and 1 slope.

C
```
PROC GLM;
     MODEL Y=X1 X2 X3 X4 X5 X6/NOINT P;
        OUTPUT OUT=NEW1 PREDICTED=YHAT1 RESIDUAL=RESID1;
```

D
```
PROC PLOT DATA=NEW1;
     PLOT RESID1*YHAT1/VREF=0;
```
Residual plot for the full model.

E
```
PROC GLM;
     MODEL Y=X1 X3 X5 TEMP/NOINT;
        OUTPUT OUT=NEW2 PREDICTED=YHAT2 RESIDUAL=RESID2;
```
Fits the reduced model.

```
PROC PLOT DATA=NEW2;
F    PLOT RESID2*YHAT2/VREF=0;
G    PLOT Y*TEMP=DAY;
```
F: The residual plot for the reduced model.

G: Y vs. TEMP, and the character used to plot each observation (Y) is the data stored in DAY which corresponds to that observation (i.e., the first letter of the day in which the observation was taken).

(A) The $\underset{\sim}{X}$ matrix for the full model in (C), p. 145

Separate Intercepts

Separate Slopes

ELECTRICITY LOAD DATA

OBS	X1	X2	X3	X4	X5	X6
1	1	77	0	0	0	0
2	1	75	0	0	0	0
3	0	0	1	74	0	0
4	0	0	0	0	1	78
5	0	0	0	0	1	77
6	1	72	0	0	0	0
7	1	79	0	0	0	0
8	1	81	0	0	0	0
9	1	85	0	0	0	0
10	0	0	1	83	0	0
11	0	0	0	0	1	82
12	1	75	0	0	0	0
13	1	77	0	0	0	0
14	1	75	0	0	0	0
15	1	77	0	0	0	0
16	1	77	0	0	0	0
17	0	0	1	82	0	0
18	0	0	0	0	1	74
19	1	72	0	0	0	0
20	1	71	0	0	0	0
21	1	73	0	0	0	0
22	1	78	0	0	0	0
23	1	78	0	0	0	0
24	0	0	1	74	0	0
25	0	0	0	0	1	77
26	1	79	0	0	0	0
27	1	73	0	0	0	0
28	1	75	0	0	0	0
29	1	69	0	0	0	0
30	1	71	0	0	0	0
31	0	0	1	68	0	0

X1 = 1 if weekday, 0 otherwise
X3 = 1 if Saturday, 0 otherwise
X5 = 1 if Sunday or holiday, 0 otherwise
X2 = temp if weekday, 0 otherwise
X4 = temp if Saturday, 0 otherwise
X6 = temp if Sunday or holiday, 0 otherwise

(B) The $\underset{\sim}{X}$ matrix for the reduced model in (E)

ELECTRICITY LOAD DATA

OBS	X1	X3	X5	TEMP
1	1	0	0	77
2	1	0	0	75
3	0	1	0	74
4	0	0	1	78
5	0	0	1	77
6	1	0	0	72
7	1	0	0	79
8	1	0	0	81
9	1	0	0	85
10	0	1	0	83
11	0	0	1	82
12	1	0	0	75
13	1	0	0	77
14	1	0	0	75
15	1	0	0	77
16	1	0	0	77
17	0	1	0	82
18	0	0	1	74
19	1	0	0	72
20	1	0	0	71
21	1	0	0	73
22	1	0	0	78
23	1	0	0	78
24	0	1	0	74
25	0	0	1	77
26	1	0	0	79
27	1	0	0	73
28	1	0	0	75
29	1	0	0	69
30	1	0	0	71
31	0	1	0	68

Separate Intercepts

Common Slope

Ⓒ FULL MODEL, 3 [illegible]RCEPTS AND 3 SLOPES

Ⓒ Y = X1 X2 X3 X4 X5 X6 NOINT

DEPENDENT VARIABLE: Y

SOURCE	DF	SUM OF SQUARES	MEAN SQUARE	F VALUE
MODEL	6	34740917.05721010	5790152.84953502	5432.09
ERROR	25	26647.90278990	1065.91611160	PR > F
UNCORRECTED TOTAL	31	34767565.00000000		0.0001

Like PROC REG, PROC GLM computes an incorrect R^2 when NOINT is used.

R-SQUARE	C.V.	STD DEV	Y MEAN
~~0.999234~~	3.1015	32.64837073	1052.67741935

SOURCE	DF	(sequentials) TYPE I SS	F VALUE	PR > F
X1	1	26243932.19047619	24621.01	0.0001
X2	1	50779.42480620	47.64	0.0001
X3	1	4723920.00000000	4431.79	0.0001
X4	1	28880.00000000	27.09	0.0001
X5	1	3692641.80000000	3464.48	0.0001
X6	1	563.68192771	0.53	0.4739

SOURCE	DF	(partials) TYPE IV SS	F VALUE	PR > F
X1	1	613.59455571	0.58	0.4551
X2	1	50779.42480620	47.64	0.0001
X3	1	103.72400562	0.10	0.7577
X4	1	28880.00000000	27.09	0.0001
X5	1	1603.84116067	1.50	0.2314
X6	1	563.68192771	0.53	0.4739

PARAMETER	PARTIAL ESTIMATE	T FOR H0: PARAMETER=0	PR > |T|	STD ERROR OF ESTIMATE
X1 weekday intercept=	110.83421927	0.76	0.4551	146.08130199
X2 weekday slope =	13.30930233	6.90	0.0001	1.92829251
X3 Satur. intercept=	-62.14285714	-0.31	0.7577	199.21039647
X4 Satur. slope =	13.57142857	5.21	0.0001	2.60728487
X5 Sunday intercept=	539.65060241	1.23	0.2314	439.94005229
X6 Sunday slope =	4.12048193	0.73	0.4739	5.66620746

PROC GLM has no option which gives estimates of the sequential b's.

Ⓔ REDUCED MODEL, 3 INTERCEPTS AND A COMMON SLOPE

Ⓔ Y = X1 X3 X5 TEMP/NOINT

GENERAL LINEAR MODELS PROCEDURE

DEPENDENT VARIABLE: Y

SOURCE	DF	SUM OF SQUARES	MEAN SQUARE	F VALUE
MODEL	4	34738249.28002397	8684562.32000599	7998.55
ERROR	27	29315.71597603	1085.76740652	PR > F
UNCORRECTED TOTAL	31	34767565.00000000		0.0001

~~R-SQUARE~~	C.V.	STD DEV	Y MEAN
~~0.999157~~	3.1302	32.95098491	1052.67741935

SOURCE	DF	Sequentials TYPE I SS	F VALUE	PR > F
X1 } intercept	1	R(1) = 26243932.19047619	24170.86	0.0001
X3 } intercept	1	R(3|1) = 4723920.00000000	4350.77	0.0001
X5 } intercept	1	R(5|3,1) = 3692841.80000000	3401.14	0.0001
TEMP ← slope	1	R(T|5,3,1) = 77555.28954779	71.43	0.0001

SOURCE	DF	Partials TYPE IV SS	F VALUE	PR > F
X1	1	R(1|3,5,T) = 1934.52471220	1.78	0.1931
X3	1	R(3|1,5,T) = 0.00006809	0.00	0.9998
X5	1	R(5|1,3,T) = 1325.61070453	1.22	0.2789
TEMP	1	R(T|1,3,5) = 77555.28954779	71.43	0.0001

PARAMETER	ESTIMATE	T FOR H0: PARAMETER=0	PR > |T|	STD ERROR OF ESTIMATE
X1 weekday intercept=	152.73674326	1.33	0.1931	114.42601814
X3 Satur. intercept=	0.02903497	0.00	0.9998	115.94506590
X5 Sunday intercept=	-130.42869930	-1.10	0.2789	118.04118545
TEMP common slope=	12.75552448	8.45	0.0001	1.50924939

Prediction equation given on p. 146

To evaluate the adequacy of the reduced model, form the F statistic

$$\frac{[(\text{Model SS, full model}) - (\text{Model SS, reduced model})]/\text{difference in df between the 2 models}}{(\text{Residual SS, full model})\,/\,(\text{Residual df, full model})}$$

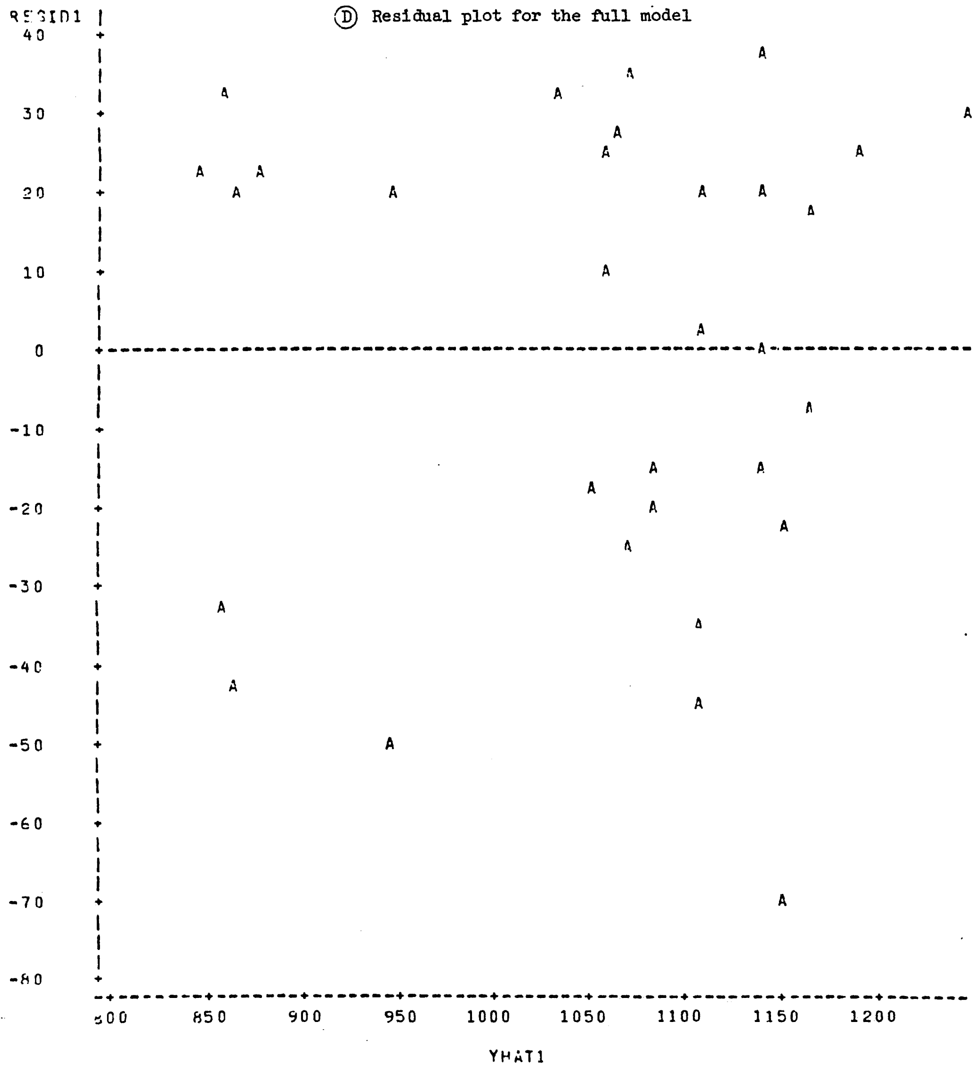
$$= \frac{[34,740,917 - 34,738,249]/2}{1065.916} \doteq 1.25 = F^2_{25}$$

Based on the relatively small F value, the reduced model is judged to be sufficient and there is no need to fit separate slopes.

RESIDUAL ANALYSIS

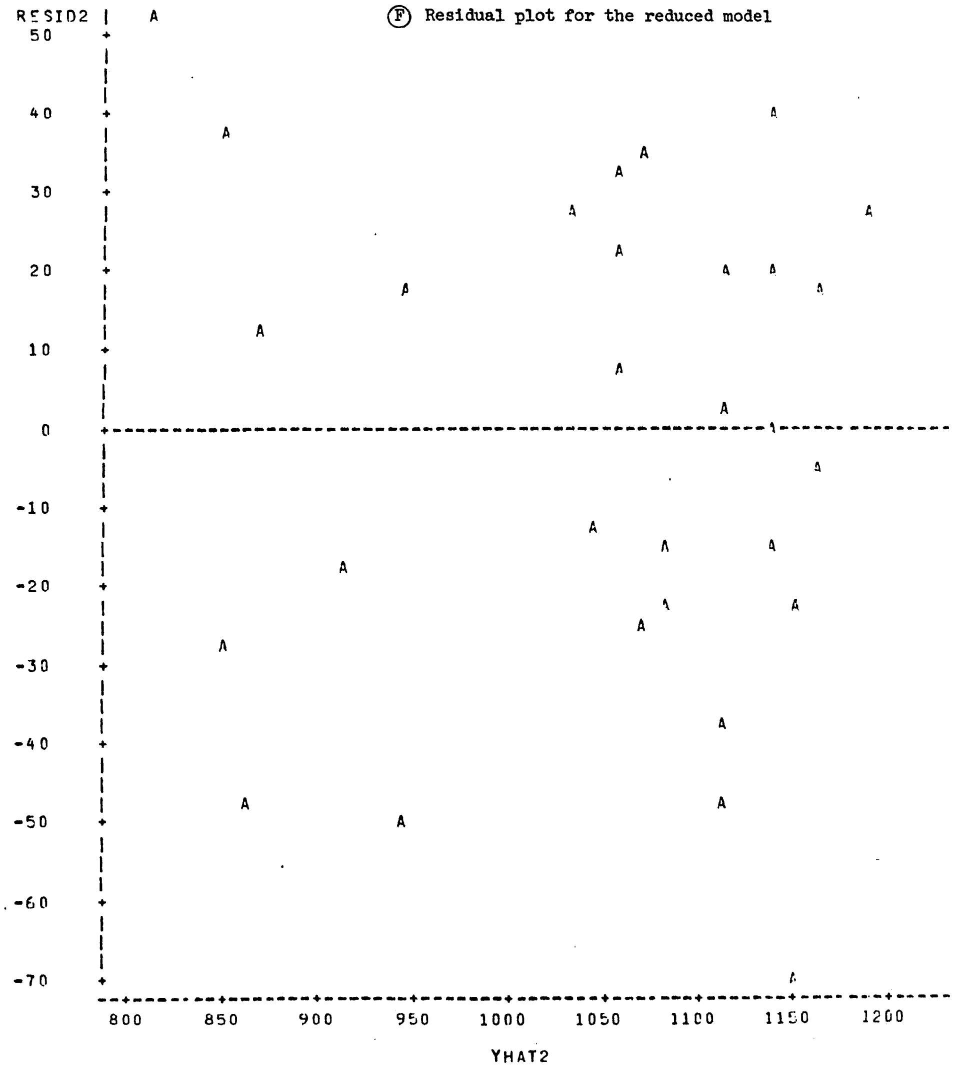
PLOT OF RESID1*YHAT1   LEGEND: A = 1 OBS, B = 2 OBS, ETC.

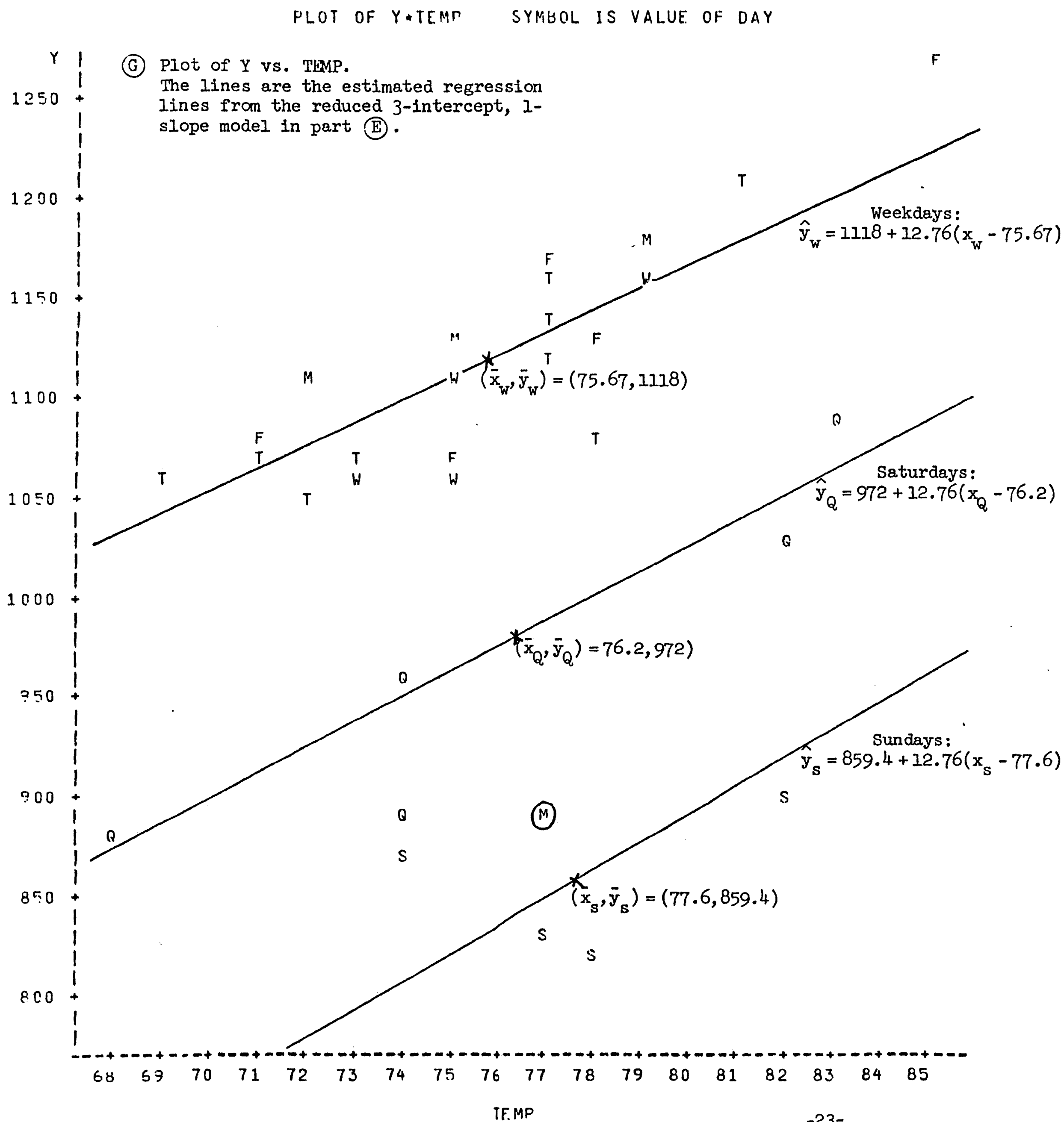
(D) Residual plot for the full model



PLOT OF RESID2*YHAT2   LEGEND: A = 1 OBS, B = 2 OBS,

(F) Residual plot for the reduced model




PLOT OF Y*TEMP   SYMBOL IS VALUE OF DAY
Y
Ⓖ Plot of Y vs. TEMP.
The lines are the estimated regression lines from the reduced 3-intercept, 1-slope model in part Ⓔ.
1250
1200
1150
1100
1050
1000
950
900
850
800
Weekdays:
$\hat{y}_w = 1118 + 12.76(x_w - 75.67)$
$(\bar{x}_w, \bar{y}_w) = (75.67, 1118)$
Saturdays:
$\hat{y}_Q = 972 + 12.76(x_Q - 76.2)$
$(\bar{x}_Q, \bar{y}_Q) = 76.2, 972)$
Sundays:
$\hat{y}_s = 859.4 + 12.76(x_s - 77.6)$
$(\bar{x}_s, \bar{y}_s) = (77.6, 859.4)$
68 69 70 71 72 73 74 75 76 77 78 79 80 81 82 83 84 85
TEMP

```
TITLE LEAFHOPPER DATA;
DATA LHOPPER;
INPUT TRTS $ DAYS;
```

DAYS is the response variable

$ indicates that TRTS is a non-numerical variable

```
CARDS;
CONTROL 2.3
CONTROL 1.7
SUCROSE 3.6
SUCROSE 4.0
GLUCOSE 3.0
GLUCOSE 2.8
FRUCTOSE 2.1
FRUCTOSE 2.3
```

The CLASS statement constructs the treatment indicator variables. CLASS is not available in PROC REG. Treatment indicators would have to be set up in the input statement as in the Electricity Load Data or the Soybean Physiological Data.

```
PROC GLM;
(A)  CLASS TRTS;
     MODEL DAYS=TRTS/NOINT SOLUTION P XPX SSI;
          OUTPUT OUT=NEW PREDICTED=YHAT RESIDUAL=RESID;
     ESTIMATE 'CONTROL VS SUGARS'
          TRTS 3 -1 -1 -1 /DIVISOR=3 E;
     ESTIMATE '6-CARBONS VS SUCROSE'
          TRTS 0 -.5 -.5 1/E;
     ESTIMATE 'FRUCTOSE VS GLUCOSE'
          TRTS 0 -1 1 0/E;
     MEANS TRTS;
```

This model, following a CLASS statement, is the equivalent of a general means model. A SOLUTION option is needed after a CLASS statement so that the parameter estimates will be printed. XPX prints the $\underset{\sim}{X}'\underset{\sim}{X}$ matrix (see pp. 179-180).

ESTIMATE contrasts, p. 181

SAS orders levels of a classed variable alphabetically, or numerically, so the coefficients must be ordered: Control, Fructose, Glucose, Sucrose.

MEANS TRTS: Calculates TRT means.

```
PROC PLOT;
(B)  PLOT RESID*YHAT/VREF=0;
```

Residual plot for the general means model in (A)

```
PROC GLM;
     CLASS TRTS;
     MODEL DAYS=TRTS/P XPX SSI;
     ESTIMATE 'CONTROL VS SUGARS'
          TRTS 3 -1 -1 -1 /DIVISOR=3 ;
     ESTIMATE '6-CARBONS VS SUCROSE'
          TRTS 0 -.5 -.5 1;
     ESTIMATE 'FRUCTOSE VS GLUCOSE'

          TRTS 0 -1 1 0;
```

```
                         LEAFHOPPER DATA

                  GENERAL LINEAR MODELS PROCEDURE

                     CLASS LEVEL INFORMATION

        CLASS      LEVELS     VALUES

        TRTS          4       CONTROL FRUCTOSE GLUCOSE SUCROSE

             NUMBER OF OBSERVATIONS IN DATA SET = 8
```

The CLASS statement produces this output.

Note alphabetical ordering

With a CLASS statement, SAS creates indicator (dummy) variables and actually forms an $\underset{\sim}{X}'\underset{\sim}{X}$ matrix as if indicator variables had been set up in the input statements.

LEAFHOPPER DATA

GENERAL LINEAR MODELS PROCEDURE

MATRIX ELEMENT REPRESENTATION

EPENDENT VARIABLE: DAYS

EFFECT		REPRESENTATION
TRTS	CONTROL	DUMMY001
	FRUCTOSE	DUMMY002
	GLUCOSE	DUMMY003
	SUCROSE	DUMMY004

LEAFHOPPER DATA

GENERAL LINEAR MODELS PROCEDURE

THE X'X MATRIX

DEPENDENT VARIABLE: DAYS

	DUMMY001	DUMMY002	DUMMY003	DUMMY004
DUMMY001	2	0	0	0
DUMMY002	0	2	0	0
DUMMY003	0	0	2	0
DUMMY004	0	0	0	2

ESTIMABLE FUNCTIONS FOR CONTROL VS SUGARS

EFFECT		COEFFICIENTS
TRTS	CONTROL	1
	FRUCTOSE	-0.333333
	GLUCOSE	-0.333333
	SUCROSE	-0.333333

Note the function of the DIVISOR = 3 option.

The E option for each ESTIMATE statement prints the contrast vector (the c vector of $c\beta$).

ESTIMABLE FUNCTIONS FOR 6 CARBONS VS SUCROSE

EFFECT		COEFFICIENTS
TRTS	CONTROL	0
	FRUCTOSE	-0.5
	GLUCOSE	-0.5
	SUCROSE	1

MEANS

TRTS	N	DAYS
CONTROL	2	2.00000000
FRUCTOSE	2	2.20000000
GLUCOSE	2	2.90000000
SUCROSE	2	3.80000000

MEANS TRTS prints the treatment means. Compare to parameter estimates on next page.

ESTIMABLE FUNCTIONS FOR FRUCTOSE VS GLUCOSE

EFFECT		COEFFICIENTS
TRTS	CONTROL	0
	FRUCTOSE	-1
	GLUCOSE	1
	SUCROSE	0

GENERAL MEANS MODEL

Ⓐ Y = TRTS/NOINT

GENERAL LINEAR MODELS PROCEDURE

DEPENDENT VARIABLE: DAYS

SOURCE	DF	SUM OF SQUARES	MEAN SQUARE	F VALUE
MODEL	4	63.38000000	15.84500000	211.27
ERROR	4	0.30000000	0.07500000	PR > F
UNCORRECTED TOTAL	8	63.68000000		0.0001

~~R-SQUARE~~	C.V.	STD DEV	DAYS MEAN
~~0.995289~~	10.0500	0.27386128	2.72500000

SOURCE	DF	TYPE I SS	F VALUE	PR > F
TRTS	4	63.38000000	211.27	0.0001 ← $H_0: \mu_C = \mu_F = \mu_G = \mu_S = 0$

PARAMETER		Sequentials from ABDO, p. 180 / ESTIMATE	T FOR H0: PARAMETER=0	PR > |T|	STD ERROR OF ESTIMATE
TRTS	CONTROL	2.00000000 $= \bar{y}_C$	10.33	0.0005	0.19364917 $= \sqrt{\frac{0.0750}{2}}$
Note order	FRUCTOSE	2.20000000 $= \bar{y}_F$	11.36	0.0003	0.19364917
	GLUCOSE	2.90000000 $= \bar{y}_G$	14.98	0.0001	0.19364917
	SUCROSE	3.80000000 $= \bar{y}_S$	19.62	0.0001	0.19364917
CONTROL VS SUGARS		-0.96666667	-4.32	0.0124	0.22360680
6-CARBONS VS SUCROSE		1.25000000	5.27	0.0062	0.23717082 Contrasts
FRUCTOSE VS GLUCOSE		0.70000000	2.56	0.0629	0.27386128

OBSERVATION	OBSERVED VALUE	PREDICTED VALUE	RESIDUAL
1	2.30000000	2.00000000	0.30000000
2	1.70000000	2.00000000	-0.30000000
3	3.60000000	3.80000000	-0.20000000
4	4.00000000	3.80000000	0.20000000
5	3.00000000	2.90000000	0.10000000
6	2.80000000	2.90000000	-0.10000000
7	2.10000000	2.20000000	-0.10000000
8	2.30000000	2.20000000	0.10000000

Control vs. Sugar contrast (see p. 181):

Estimate = -.9667 = 2.0 - .3333(2.2) - .3333(2.9) - .3333(3.8)

Standard Error= .2236 = SQRT[$\frac{\sigma^2}{2}$ (1+1/9+1/9+1/9)]

GENERAL MEANS MODEL

(C) Y = TRTS

LEAFHOPPER DATA

GENERAL LINEAR MODELS PROCEDURE

DEPENDENT VARIABLE: DAYS

SOURCE	DF	SUM OF SQUARES	MEAN SQUARE	F VALUE
MODEL	3	3.97500000	1.32500000	17.67
ERROR	4	0.30000000	0.07500000	PR > F
CORRECTED TOTAL	7	4.27500000		0.0090

$H_0: \mu_C = \mu_F = \mu_G = \mu_S = \mu$

R-SQUARE	C.V.	STD DEV	DAYS MEAN
0.929825	10.0500	0.27386128	2.72500000

SOURCE	DF	TYPE I SS	F VALUE	PR > F
TRTS	3	3.97500000	17.67	0.0090

PARAMETER	ESTIMATE	T FOR H0: PARAMETER=0	PR > |T|	STD ERROR OF ESTIMATE
CONTROL VS SUGARS	-0.96666667	-4.32	0.0124	0.22360680
6-CARBONS VS SUCROSE	1.25000000	5.27	0.0062	0.23717082
FRUCTOSE VS GLUCOSE	0.70000000	2.56	0.0629	0.27386128

OBSERVATION	OBSERVED VALUE	PREDICTED VALUE	RESIDUAL
1	2.30000000	2.00000000	0.30000000
2	1.70000000	2.00000000	-0.30000000
3	3.60000000	3.80000000	-0.20000000
4	4.00000000	3.80000000	0.20000000
5	3.00000000	2.90000000	0.10000000
6	2.80000000	2.90000000	-0.10000000
7	2.10000000	2.20000000	-0.10000000
8	2.30000000	2.20000000	0.10000000

SUM OF RESIDUALS	0.00000000
SUM OF SQUARED RESIDUALS	0.30000000
SUM OF SQUARED RESIDUALS - ERROR SS	-0.00000000
FIRST ORDER AUTOCORRELATION	-0.20000000
DURBIN-WATSON D	2.06666667

LEAFHOPPER DATA

GENERAL LINEAR MODELS PROCEDURE

CLASS LEVEL INFORMATION

CLASS	LEVELS	VALUES
TRTS	4	CONTROL FRUCTOSE GLUCOSE SUCROSE

NUMBER OF OBSERVATIONS IN DATA SET = 8

	no stimulation	anti-IG stimulation
no ATP	treatment (0,0)	treatment (0,1)
ATP added	treatment (1,0)	treatment (1,1)

```
TITLE LYMPHOCYTE DATA;
DATA LYMPH;
   INPUT ATP IG Y;
   CARDS;
```

```
    PROC GLM;
(A)   CLASSES ATP IG; <------------- Class on both ATP and IG.
      MODEL Y = ATP IG ATP*IG/P;           } This model fits the main effects of ATP and IG,
        OUTPUT OUT = NEW   R=RESID  P=YHAT; } and their interaction, ATP*IG.
```

```
    PROC GLM DATA = LYMPH;
(B)   CLASSES ATP IG;
      MODEL Y = ATP*IG/NOINT SOLUTION SS1;
        ESTIMATE 'STIMULUS' ATP*IG 1 -1 1 -1/DIVISOR=2;
        ESTIMATE 'ATP PRESENCE' ATP*IG 1 1 -1 -1/DIVISOR=2;
        ESTIMATE 'INTERACTION'  ATP*IG 1 -1 -1 1/DIVISOR=2;
        ESTIMATE 'INTERACTION ADJ' ATP*IG 1 -1 -1 1;
                                    |
                                    ---> Refers to variables in CLASSES Statement.
```

Use the general means model to estimate contrasts.
Contrasts among treatment means in part (A) would be "non-estimable" because the main effects are fitted first.

The CLASSES statement orders the treatments numerically: (0,0); (0,1); (1,0); (1,1). The contrast coefficients must be in this order.

```
    PROC PLOT;
(C)   PLOT RESID*YHAT/VREF=0; <--- Residual plot for (A) and (B).
```

FACTORIAL ANALYSIS
(A) Y = ATP IG ATP*IG

DEPENDENT VARIABLE: Y

SOURCE	DF	SUM OF SQUARES	MEAN SQUARE	F VALUE
MODEL	3	1906.00000000	635.33333333	87.63
ERROR	4	29.00000000	7.25000000	PR > F
CORRECTED TOTAL	7	1935.00000000		0.0004

R-SQUARE	C.V.	STD DEV	Y MEAN
0.985013	6.3355	2.69258240	42.50000000

SOURCE	DF	TYPE I SS	F VALUE	PR > F
ATP	1	288.00000000	39.72	0.0032
IG	1	1568.00000000	216.28	0.0001
ATP*IG	1	50.00000000	6.90	0.0584

SOURCE	DF	TYPE IV SS	F VALUE	PR > F
ATP	1	288.00000000	39.72	0.0032
IG	1	1568.00000000	216.28	0.0001
ATP*IG	1	50.00000000	6.90	0.0584

Note that TYPE I and TYPE IV SS's are the same due to orthogonality. The SSI option could have been used.

OBSERVATION	OBSERVED VALUE	PREDICTED VALUE	RESIDUAL
1	34.00000000	32.00000000	2.00000000
2	30.00000000	32.00000000	-2.00000000
3	62.50000000	65.00000000	-2.50000000
4	67.50000000	65.00000000	2.50000000
5	25.50000000	25.00000000	0.50000000
6	24.50000000	25.00000000	-0.50000000
7	46.00000000	48.00000000	-2.00000000
8	50.00000000	48.00000000	2.00000000

SUM OF RESIDUALS	0.00000000
SUM OF SQUARED RESIDUALS	29.00000000
SUM OF SQUARED RESIDUALS - ERROR SS	0.00000000
~~FIRST ORDER AUTOCORRELATION~~	~~-0.25000000~~
~~DURBIN-WATSON D~~	~~2.22413793~~

GENERAL MEANS MODEL

(B) Y = ATP * IG/NOINT

GENERAL LINEAR MODELS PROCEDURE

DEPENDENT VARIABLE: Y

SOURCE	DF	SUM OF SQUARES	MEAN SQUARE	F VALUE	
MODEL	4	16356.00000000	4089.00000000	564.00	
ERROR	4	29.00000000	7.25000000 = s^2	PR > F	s^2 same as for (A)
UNCORRECTED TOTAL	8	16385.00000000		0.0001	

~~R-SQUARE~~	C.V.	STD DEV	Y MEAN
~~0.998230~~	6.3355	2.69258240	42.50000000

SOURCE	DF	TYPE I SS	F VALUE	PR > F
ATP*IG	4	16356.00000000	564.00	0.0001

PARAMETER		ESTIMATE		T FOR H0: PARAMETER=0	PR > |T|	STD ERROR OF ESTIMATE
ATP*IG	0 0	32.00000000 = $\bar{y}_{00}$		16.81	0.0001	1.90394328
	0 1	65.00000000 = $\bar{y}_{01}$		34.14	0.0001	1.90394328
	1 0	25.00000000 = $\bar{y}_{10}$		13.13	0.0002	1.90394328
	1 1	48.00000000 = $\bar{y}_{11}$		25.21	0.0001	1.90394328
STIMULUS		-28.00000000	p. 197	-14.71	0.0001	1.90394328
ATP PRESENCE		12.00000000		6.30	0.0032	1.90394328
INTERACTION		-5.00000000		-2.63	0.0584	1.90394328
INTERACTION ADJ		**-10.00000000**		**-2.63**	**0.0584**	**3.80788655**

Compare significance levels to those of the TYPE I SS in part (A)

In (A) the contrasts are computed as part of the model.
In (B) the contrasts are specified.

```
TITLE FAT DIGESTIBILITY DATA;
DATA FAT_DIG;
     INPUT BLOCK FAT $ LECITHIN Y;
     LABEL BLOCK = PERIOD;
     IF FAT='T' AND LECITHIN=0 THEN  TMT=1;
       ELSE IF FAT='C' AND LECITHIN=0 THEN  TMT=2;
         ELSE IF FAT='T' AND LECITHIN=1 THEN  TMT=3;
           ELSE IF FAT='C' AND LECITHIN=1 THEN  TMT=4;
     CARDS;
T 0 64.6
T 0 52.4
T 0 53.8
C 0 66.0
C 0 60.1
C 0 64.4
T 1 85.0
T 1 68.9
T 1 77.5
C 1 96.0
C 1 90.4
C 1 98.2
```

Creating the indicator variables.

```
 PROC GLM;
  CLASS BLOCK TMT;
  MODEL Y = BLOCK TMT/XPX;
A OUTPUT OUT=NEW R=RESID P=YHAT;
   ESTIMATE 'W VS WO LECITHIN' TMT .5 .5 -.5 -.5;
   ESTIMATE 'FAT DIFF WO LECITHIN' TMT  1 -1 0 0;
   ESTIMATE 'FAT DIFF W LECITHIN'  TMT  0  0  1 -1;
```

Model: Equal means|Period Indicators|Treatment Indicators and contrasts as discussed in Unit 17.

```
 PROC PLOT;
B PLOT RESID*YHAT/VREF=0;
```

Residual plot for A.

GENERAL LINEAR MODELS PROCEDURE

CLASS LEVEL INFORMATION

CLASS	LEVELS	VALUES
BLOCK	3	1 2 3
TMT	4	1 2 3 4

NUMBER OF OBSERVATIONS IN DATA SET = 12

MODEL SEQUENCE

(A) Y = Equal means | Periods | Treatments

THE X'X MATRIX

DEPENDENT VARIABLE: Y

	INTERCEPT	BLOCK 1	BLOCK 2	BLOCK 3	TMT 1	TMT 2	TMT 3	TMT 4
INTERCEPT	12	4	4	4	3	3	3	3
BLOCK 1	4	4	0	0	1	1	1	1
BLOCK 2	4	0	4	0	1	1	1	1
BLOCK 3	4	0	0	4	1	1	1	1
TMT 1	3	1	1	1	3	0	0	0
TMT 2	3	1	1	1	0	3	0	0
TMT 3	3	1	1	1	0	0	3	0
TMT 4	3	1	1	1	0	0	0	3

DEPENDENT VARIABLE: Y

SOURCE	DF	SUM OF SQUARES	MEAN SQUARE	F VALUE
MODEL	5	2729.54083333	545.90816667	46.06
ERROR	6	71.10833333	11.85138889	PR > F
CORRECTED TOTAL	11	2800.64916667		0.0001

R-SQUARE	C.V.	ROOT MSE	Y MEAN
0.974610	4.7089	3.44258462	73.10833333

SOURCE	DF	TYPE I SS	F VALUE	PR > F
BLOCK	2	198.81166667	8.39	0.0183
TMT	3	2530.72916667	71.18	0.0001

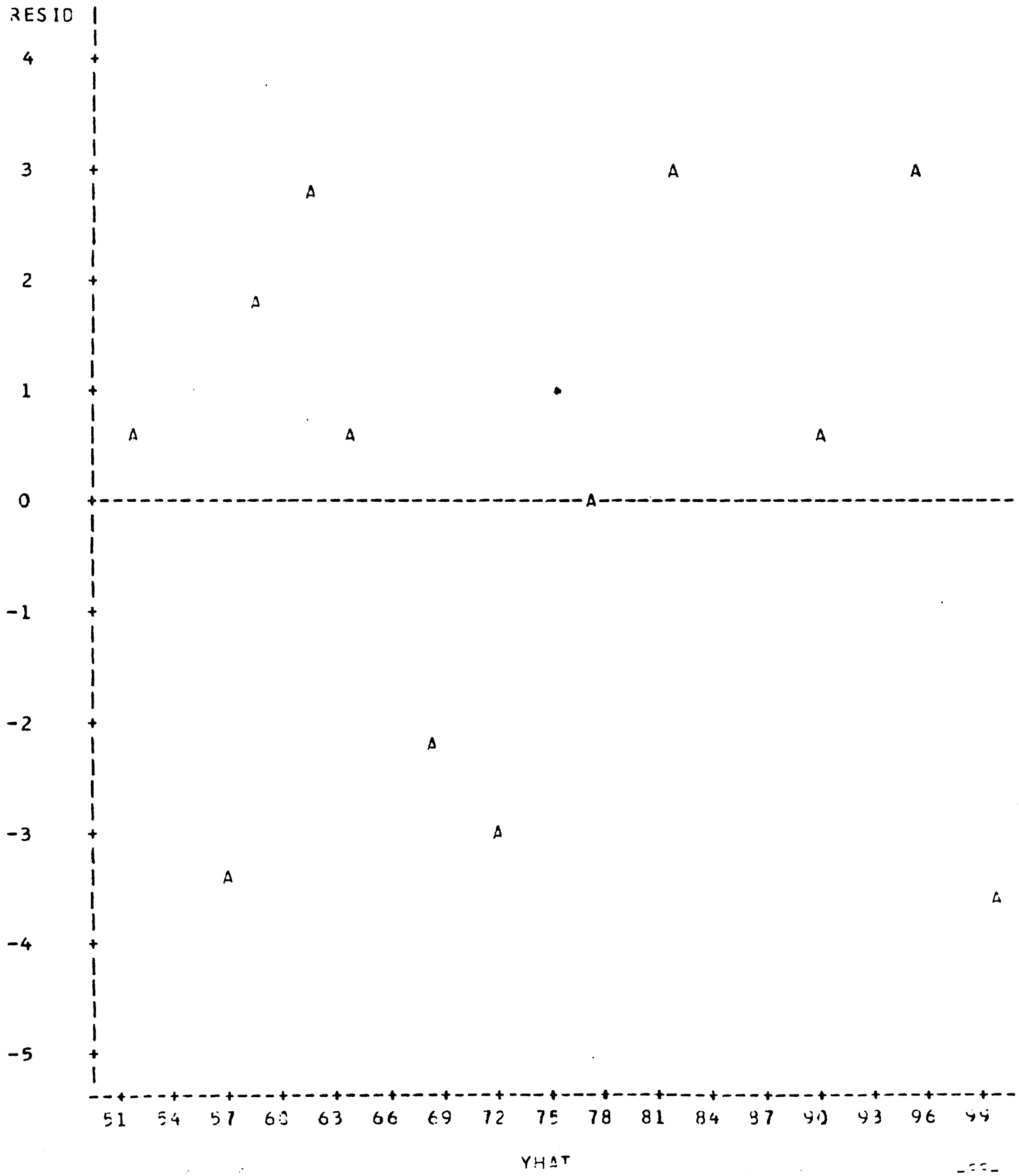
SOURCE	DF	TYPE III SS	F VALUE	PR > F
BLOCK	2	198.81166667	8.39	0.0183
TMT	3	2530.72916667	71.18	0.0001

PARAMETER	ESTIMATE	T FOR H0: PARAMETER=0	PR > |T|	STD ERROR OF ESTIMATE
Contrasts:				
W VS WO LECITHIN	-25.78333333	-12.97	0.0001	1.98757716
FAT DIFF WO LECITHIN	-6.56666667	-2.34	0.0581	2.81085857
FAT DIFF W LECITHIN	-17.73333333	-6.31	0.0007	2.81085857

The XPX option on the model statement produces this listing of the X'X matrix. Notice that the blocks are orthogonal to one another (upper left box) and the treatments are orthogonal to one another (lower right box). Further, the blocks are orthogonal to the treatments.

FAT DIGESTIBILITY DATA (B) Residual plot from (E).

PLOT OF RESID*YHAT LEGEND: A = 1 OBS, B = 2 OBS, ETC.



```
TITLE NUTRITION DATA;
DATA PROTEIN;
  INPUT PROTEIN $ Y;
CARDS;
H 179
H 136
H 160
H 227
H 217     10 horsebean observations
H 168
H 108
H 124
H 143
H 140
L 309
L 229
L 181
L 141
L 260
L 203     12 linseed observations
L 148
L 169
L 213
L 257
L 244
L 271
S 243
S 230
S 248
S 327
S 329
S 250
S 193     14 soybean observations
S 271
S 316
S 267
S 199
S 177
S 158
S 248
```

See p. 217
Natural Contrasts

```
 PROC GLM;
Ⓐ  CLASS PROTEIN;
   MODEL Y = PROTEIN/NOINT SOLUTION P SS1;
     OUTPUT OUT=NEW P=YHAT R=RESID;
    ESTIMATE 'HORSEBEAN VS OILMEAL' PROTEIN 1 -.5 -.5;
    ESTIMATE 'LINSEED VS SOYBEAN'   PROTEIN 0 1 -1;
    ESTIMATE 'ORTHO H-B VS OILMEAL' PROTEIN 13 -6 -7/DIVISOR=13;
```

Using the CLASS statement to fit the general means model, followed by natural and orthogonal contrasts, as discussed in Unit 18.

See p. 219
Ortho Contrasts

```
 PROC PLOT;
Ⓑ  PLOT RESID*YHAT/VREF=0; Residual plot for Ⓐ.
```

GENERAL MEANS MODEL

(A) Y = PROTEIN/NOINT

DEPENDENT VARIABLE: Y

SOURCE	DF	SUM OF SQUARES	MEAN SQUARE	F VALUE
MODEL	3	1683997.43571429	561332.47857143	229.66
ERROR	33	80659.56428571	2444.22922078	PR > F
UNCORRECTED TOTAL	36	1764657.00000000		0.0001

R-SQUARE	C.V.	STD DEV	Y MEAN
0.954292	23.1656	49.43914664	213.41666667

SOURCE	DF	TYPE I SS	F VALUE	PR > F
PROTEIN	3	1683997.43571429	229.66	0.0001

← Tests: $\mu_H = \mu_L = \mu_S = 0$ not $\mu_H = \mu_L = \mu_S = \mu$

PARAMETER		ESTIMATE	T FOR H0: PARAMETER=0	PR > |T|	STD ERROR OF ESTIMATE
PROTEIN	H	160.20000000 = $\bar{y}_H$	10.25	0.0001	15.63403090
	L	218.75000000 = $\bar{y}_L$	15.33	0.0001	14.27185231
	S	246.85714286 = $\bar{y}_S$	18.68	0.0001	13.21316773
Contrasts:					
HORSEBEAN VS OILMEAL		-72.60357143	-3.94	0.0004	18.41171677
LINSEED VS SOYBEAN		-28.10714286	-1.45	0.1578	19.44925628
ORTHO H-B VS OILMEAL		-73.68461538	-4.01	0.0003	18.39651430

In this case, the natural and ORTHO contrasts give similar results.

OBSERVATION	OBSERVED VALUE	PREDICTED VALUE	RESIDUAL
1	179.00000000	160.20000000	18.80000000
2	136.00000000	160.20000000	-24.20000000
3	160.00000000	160.20000000	-0.20000000
4	227.00000000	160.20000000	66.80000000
5	217.00000000	160.20000000	56.80000000
6	168.00000000	160.20000000	7.80000000
7	108.00000000	160.20000000	-52.20000000
8	124.00000000	160.20000000	-36.20000000
9	143.00000000	160.20000000	-17.20000000
10	140.00000000	160.20000000	-20.20000000
11	309.00000000	218.75000000	90.25000000
12	229.00000000	218.75000000	10.25000000
13	181.00000000	218.75000000	-37.75000000
14	141.00000000	218.75000000	-77.75000000
15	260.00000000	218.75000000	41.25000000
16	203.00000000	218.75000000	-15.75000000
17	148.00000000	218.75000000	-70.75000000
18	169.00000000	218.75000000	-49.75000000
19	213.00000000	218.75000000	-5.75000000
20	257.00000000	218.75000000	38.25000000

OBSERVED VALUE	PREDICTED VALUE	RESIDUAL
244.00000000	218.75000000	25.25000000
271.00000000	218.75000000	52.25000000
243.00000000	246.85714286	-3.85714286
230.00000000	246.85714286	-16.85714286
248.00000000	246.85714286	1.14285714
327.00000000	246.85714286	80.14285714
329.00000000	246.85714286	82.14285714
250.00000000	246.85714286	3.14285714
193.00000000	246.85714286	-53.85714286
271.00000000	246.85714286	24.14285714
316.00000000	246.85714286	69.14285714
267.00000000	246.85714286	20.14285714
199.00000000	246.85714286	-47.85714286
177.00000000	246.85714286	-69.85714286
158.00000000	246.85714286	-88.85714286
248.00000000	246.85714286	1.14285714

SUM OF RESIDUALS	0.00000000
SUM OF SQUARED RESIDUALS	80659.56428571
SUM OF SQUARED RESIDUALS - ERROR SS	-0.00000000
FIRST ORDER AUTOCORRELATION	0.34539716
DURBIN-WATSON D	1.30480762

```
OPTIONS LS=75 NODATE;
DATA SWAMP;
  INPUT LOC TYPE TMT Y;
 CARDS;
```

Creates the data set SWAMP.

```
PROC GLM;
  CLASSES LOC TYPE;
(A) MODEL Y=LOC TYPE LOC*TYPE/P SS1 SS2 SS3 CLM;
  OUTPUT OUT=NEW P=YHAT R=RESID1;
   MEANS LOC TYPE LOC*TYPE / DEPONLY;
   LSMEANS LOC TYPE LOC*TYPE / STDERR;
```

```
PROC GLM;
  CLASSES LOC TYPE;
  MODEL Y=LOC*TYPE/NOINT;
(B) ESTIMATE 'CONTRAST1 IN ROW1' LOC*TYPE 0 1 -1 0 0 0;
(C) ESTIMATE 'WEIGHTED CONT2 ROW1' LOC*TYPE 11 -6 -5 0 0 0/DIVISOR=11;
(B) ESTIMATE 'CONTRAST1 IN ROW2' LOC*TYPE 0 0 0 0 1 -1;
(C) ESTIMATE 'WEIGHTED CONT2 ROW2' LOC*TYPE 0 0 0 14 -8 -6/DIVISOR=14;

(D) ESTIMATE 'UNWGHTED CONT2 ROW1' LOC*TYPE 2 -1 -1 0 0 0/DIVISOR=2;
(D) ESTIMATE 'UNWGHTED CONT2 ROW2' LOC*TYPE 0 0 0 2 -1 -1/DIVISOR=2;
```

```
PROC GLM;
  CLASSES LOC TYPE;
(E) MODEL Y=LOC TYPE/SS1 SS2;
  ESTIMATE 'LOC NAT MAIN EFFECT ' LOC 1 -1;
  ESTIMATE 'TYPE NAT MAIN EFECT1' TYPE 0 1 -1;
(F) ESTIMATE 'TYPE NAT MAIN EFECT2' TYPE 1 -.5 -.5;

(G) ESTIMATE 'TYPE PRO MAIN EFECT2' TYPE 25 -14 -11/DIVISOR=25;

(H) ESTIMATE 'TYPE ORTHO MAIN EFF2' TYPE 1 -.552577 -.447423;
```

Flow Chart of the Analysis

1. (A) Model Sequence for:
   a. composite test of interaction
   b. residual evaluation.
2. Decide if interaction is important.

Interaction is important

(B) 3. Make 1^{st} simple effects contrasts in each row (or col.).

4. Decide if 2^{nd} simple contrasts should be weighted or unweighted.

↓ (C) ↓ (D)

Interaction is not important.

(E) 3. Force interaction into the error term.

Choose among

(F) — Natural main effect contrasts.

(G) — Proportional main effect contrasts

(H) — 2^{nd} main effect contrast orthogonal to 1^{st} main effect contrast (adjusted for unequal n_{ij}).

pH DATA

(A) General Means Model Analysis Using Model Sequence

GENERAL LINEAR MODELS PROCEDURE

DEPENDENT VARIABLE: Y

SOURCE	DF	SUM OF SQUARES	MEAN SQUARE	F VALUE
MODEL	5	2.42538095	0.48507619	2.50
ERROR	29	5.63633333	0.19435632	PR > F
CORRECTED TOTAL	34	8.06171429		0.0536

R-SQUARE	C.V.	ROOT MSE	Y MEAN
0.300852	6.6167	0.44085862	6.66285714

R(Mean) = $(35)(6.66285)^2$ = 1553.8

SOURCE	DF	TYPE I SS	F VALUE	PR > F
LOC	1	0.77487218	3.99	0.0553
TYPE	2	1.04276977	2.68	0.0853
LOC*TYPE	2	0.60773900	1.56	0.2266

1.8176 = 0.77487218 + 1.04276977

SOURCE	DF	$\boxed{\text{TYPE II SS}}$	F VALUE	PR > F
LOC	1	1.33577182	6.87	0.0138
TYPE	2	1.04276977	2.68	0.0853
LOC*TYPE	2	0.60773900	1.56	0.2266

LOC|TYPE TYPE|LOC

SOURCE	DF	TYPE III SS	F VALUE	PR > F
LOC	1	1.00971645	5.20	0.0302
TYPE	2	0.50903840	1.31	0.2854
LOC*TYPE (Interaction)	2	0.60773900	1.56	0.2266

ANOVA (1)

Source	df	SS
R(Mean)	1	1553.8
R(LOC)	1	0.7749
R(TYPE|LOC)	2	1.043
R(LOC*TYPE|LOC,TYPE)	2	0.6077
Residual	29	5.636

ANOVA (2)

Source	df	SS
R(Mean)	1	1553.8
R(TYPE)	2	1.8176 - 1.3358 = 0.4818
R(LOC|TYPE)	1	1.335
R(INTERACTION)	2	0.6077
Residual	29	

(1) = [6.825(8) + 6.467(6) + 6.12(5)]/19 – (weighted average of observations)

(2) = [6.825(8) + 6.9(2)]/10

(3) = [6.757 + 6.391 + 6.319]/3 – (unweighted average of estimated cell means)

(4) = [6.757 + 7.172]/2

		MEANS	LEAST SQUARES MEANS	
LOC	N	Y	Y LSMEAN	STD ERR LSMEAN
0 Near	19	(1) 6.52631579	(3) 6.47055556	0.10304181
1 Away	16	6.82500000	6.85000000	0.13075228

TYPE	N	Y	Y LSMEAN	STD ERR LSMEAN
0 North	10	(2) 6.84000000	(4) 6.86250000	0.17426467
1 Mesic	14	6.62857143	6.60833333	0.11904543
2 Shrub	11	6.54545455	6.51000000	0.13347658

LOC	TYPE	N	Y	Y LSMEAN	STD ERR LSMEAN
0 Near	0 North	8	6.82500000	6.82500000	0.15586706
0	1 Mesic	6	6.46666667	6.46666667	0.17997978
0	2 Shrub	5	6.12000000	6.12000000	0.19715797
1 Away	0 North	2	6.90000000	6.90000000	0.31173412
1	1 Mesic	8	6.75000000	6.75000000	0.15586706
1	2 Shrub	6	6.90000000	6.90000000	0.17997978

Notice that the cell means agree with the cell least square means.
For the General Means model the predicted values are these cell means.

If interaction is judged important ⇒ (B) then (C) or (D).

If interaction is not judged to be important
⇒ both LOC and TYPE are needed (pg. 230) ⇒ (E) then (F), (G), or (H).

Residual Analysis:

Not shown but the magnitude of the residuals is acceptable and no pattern is evident (one or two values should be checked).

(E) General Means Estimation

SAS

GENERAL LINEAR MODELS PROCEDURE

DEPENDENT VARIABLE: Y

SOURCE	DF	SUM OF SQUARES	MEAN SQUARE	F VALUE
MODEL	6	1556.20366667	259.36727778	1334.49
ERROR	29	5.63633333	0.19435632	PR > F
UNCORRECTED TOTAL	35	1561.84000000		0.0001

Interaction is judged to be important (see text p. 220 and 229).

Objective: (1) to estimate the cell means from the general means model;

(2) to estimate column contrasts within each row (or row contrasts within each column).

R-SQUARE	C.V.	ROOT MSE	Y MEAN
0.996391	6.6167	0.44085862	6.66285714

SOURCE	DF	TYPE I SS	F VALUE	PR > F
LOC*TYPE	6	1556.20366667	1334.49	0.0001

SOURCE	DF	TYPE III SS	F VALUE	PR > F
LOC*TYPE	6	1556.20366667	1334.49	0.0001

PARAMETER	ESTIMATE	T FOR H0: PARAMETER=0	PR > |T|	STD ERROR OF ESTIMATE
(E) CONTRAST1 IN ROW1	0.34666667	1.30	0.2043	0.26695315
(C) WEIGHTED CONT2 ROW1	0.51590909	2.52	0.0176	0.20484945
(E) CONTRAST1 IN ROW2	-0.15000000	-0.63	0.5336	0.23809087
(C) WEIGHTED CONT2 ROW2	0.08571429	0.26	0.7988	0.33325779
(U) UNWGHTED CONT2 ROW1	0.53166667	2.59	0.0148	0.20520852
(U) UNWGHTED CONT2 ROW2	0.07500000	0.22	0.8237	0.33369144

Interpretation: (1) Within the near location, community type North has a higher (0.5) pH than the average of the other two community types (p = .02).

(2) There is some evidence that within the near location, the pH for the Mesic Community is higher than Shrub Community (p = .20).

(3) Within the away location, the pH for community types does not vary.

(E) General Means Esti...tion (with No Interaction)

SAS

GENERAL LINEAR MODELS PROCEDURE

DEPENDENT VARIABLE: Y

SOURCE	DF	SUM OF SQUARES	MEAN SQUARE	F VALUE
MODEL	3	1.81764195	0.60588065	3.01
ERROR	31	6.24407234	0.20142169	PR > F
CORRECTED TOTAL	34	8.06171429		0.0451

R-SQUARE	C.V.	ROOT MSE	Y MEAN
0.225466	6.7359	0.44880028	6.66285714

SOURCE	DF	TYPE I SS	F VALUE	PR > F
LOC	1	0.77487218	3.85	0.0589
TYPE	2	1.04276977	2.59	0.0913

SOURCE	DF	TYPE II SS	F VALUE	PR > F
LOC	1	1.33577182	6.63	0.0150
TYPE	2	1.04276977	2.59	0.0913

	PARAMETER	ESTIMATE	T FOR H0: PARAMETER=0	PR > |T|	STD ERROR OF ESTIMATE		
	LOC NAT MAIN EFFECT	-0.41500335	-2.58	0.0150	0.16115307	1	6.904 - 6.489 = 0.415
	TYPE NAT MAIN EFECT1	0.07233758	0.40	0.6919	0.18087522	3	6.599 - 6.527 = 0.072
F	TYPE NAT MAIN EFECT2	0.40174146	2.26	0.0311	0.17791409	2	6.965 - [6.599 + 6.527]/2 = 0.4017
G	TYPE PRO MAIN EFECT2	0.39740121	2.24	0.0326	0.17766482	5	[(25)(6.965) - (14)(6.599) - (11)(6.527)]/25 = 0.397
H	TYPE ORTHO MAIN EFF2	0.39793817	2.24	0.0324	0.17765975	4	6.965 - [(.552577)(6.599) + (.447423)(6.527)] = 0.398

Situation: The interaction is judged to be unimportant but both LOC and TYPE are needed in the model.

Objective: To estimate main effects contrasts from the restricted model. The restricted model (without interaction) both adds the interaction SS to the error SS AND forces differences between rows (cols.) within all columns (rows) to be equal.

We can see these restricted means with Means, LS Means or the P option.

Community

LOC	North	Mesic	Shrub
near	6.757	6.391	6.319
away	7.172	6.806	6.734

Note that the difference between rows is the same for each column (any contrast among columns is the same for each row).

These main effect contrasts are contrasts among the restricted cell means.

OBSERVATION	OBSERVED	PREDICTED	RESIDUAL	LOWER 95% CLM	UPPER 95% CLM
1	6.60000000	6.82500000	-0.22500000	6.50621858	7.14378142
2	7.20000000	6.82500000	0.37500000	6.50621858	7.14378142
3	7.20000000	6.82500000	0.37500000	6.50621858	7.14378142
4	7.00000000	6.82500000	0.17500000	6.50621858	7.14378142
5	6.80000000	6.82500000	-0.02500000	6.50621858	7.14378142
6	6.40000000	6.82500000	-0.42500000	6.50621858	7.14378142
7	7.00000000	6.82500000	0.17500000	6.50621858	7.14378142
8	6.40000000	6.82500000	-0.42500000	6.50621858	7.14378142
9	6.80000000	6.46666667	0.33333333	6.09856959	6.83476374
10	7.00000000	6.46666667	0.53333333	6.09856959	6.83476374
11	6.20000000	6.46666667	-0.26666667	6.09856959	6.83476374
12	6.20000000	6.46666667	-0.26666667	6.09856959	6.83476374
13	6.40000000	6.46666667	-0.06666667	6.09856959	6.83476374
14	6.20000000	6.46666667	-0.26666667	6.09856959	6.83476374
15	6.40000000	6.12000000	0.28000000	5.71676985	6.52323015
16	5.20000000	6.12000000	-0.92000000	5.71676985	6.52323015
17	6.20000000	6.12000000	0.08000000	5.71676985	6.52323015
18	6.40000000	6.12000000	0.28000000	5.71676985	6.52323015
19	6.40000000	6.12000000	0.28000000	5.71676985	6.52323015
20	6.80000000	6.90000000	-0.10000000	6.26243716	7.53756284
21	7.00000000	6.90000000	0.10000000	6.26243716	7.53756284
22	6.20000000	6.75000000	-0.55000000	6.43121858	7.06878142
23	5.60000000	6.75000000	-1.15000000	6.43121858	7.06878142
24	7.20000000	6.75000000	0.45000000	6.43121858	7.06878142
25	7.20000000	6.75000000	0.45000000	6.43121858	7.06878142
26	7.20000000	6.75000000	0.45000000	6.43121858	7.06878142
27	6.20000000	6.75000000	-0.55000000	6.43121858	7.06878142
28	7.20000000	6.75000000	0.45000000	6.43121858	7.06878142
29	7.20000000	6.75000000	0.45000000	6.43121858	7.06878142
30	7.20000000	6.90000000	0.30000000	6.53190292	7.26809708
31	6.80000000	6.90000000	-0.10000000	6.53190292	7.26809708
32	7.00000000	6.90000000	0.10000000	6.53190292	7.26809708
33	6.80000000	6.90000000	-0.10000000	6.53190292	7.26809708
34	7.00000000	6.90000000	0.10000000	6.53190292	7.26809708
35	6.60000000	6.90000000	-0.30000000	6.53190292	7.26809708

SUM OF RESIDUALS	0.00000000
SUM OF SQUARED RESIDUALS	5.63633333
SUM OF SQUARED RESIDUALS - ERROR SS	0.00000000
PRESS STATISTIC	7.80612245
FIRST ORDER AUTOCORRELATION	-0.02583880
DURBIN-WATSON D	2.02672788

SWAMP DATA - Unweighted Analysis of Cell Means
see Snedecor & Cochran, 7 ed., p. 418

```
DATA SWAMP;
INPUT LOC TYPE TMT Y;
CARDS;
```

```
PROC ANOVA; CLASS TMT;
     MODEL  Y = TMT;
     MEANS TMT / DEPONLY;
```

(I) The six treatment combinations are indicated in the CLASS variable TMT. The residual MS is an estimate of σ^2.

```
PROC SORT; BY TYPE LOC;
```

```
PROC MEANS MEAN NOPRINT; BY TYPE LOC;
     OUTPUT OUT=NEW MEAN=MY; VAR Y;
```

(J) The six cell means are computed and placed in the data set NEW.

```
PROC ANOVA; CLASS TYPE LOC;
     MODEL  MY = LOC TYPE LOC*TYPE;
     MEANS LOC TYPE LOC*TYPE / DEPONLY;
```

(K) The main effect SS and interaction SS are determined at this step.

Ⓘ One-way ANOVA on the six treatment combinations.

ANALYSIS OF VARIANCE PROCEDURE

CLASS LEVEL INFORMATION

CLASS	LEVELS	VALUES
TMT	6	1 2 3 4 5 6

NUMBER OF OBSERVATIONS IN DATA SET = 35

DEPENDENT VARIABLE: Y

SOURCE	DF	SUM OF SQUARES	MEAN SQUARE	F VALUE
MODEL	5	2.42538095	0.48507619	2.50
ERROR	29	5.63633333	$\boxed{0.19435632}$	PR > F
CORRECTED TOTAL	34	8.06171429	$= s^2$	0.0536

Note: s^2 is all that is to be used from this ANOVA table along with the associated error df = 29 .

R-SQUARE	C.V.	STD DEV	Y MEAN
0.300852	6.6167	0.44085862	6.66285714

SOURCE	DF	ANOVA SS	F VALUE	PR > F
TMT	5	2.42538095	2.50	0.0536

MEANS are the cell means

TMT	N	Y
1	8	6.82500000
2	6	6.46666667
3	5	6.12000000
4	2	6.90000000
5	8	6.75000000
6	6	6.90000000

Note: Since $\frac{\max(n_{ij})}{\min(n_{ij})} = \frac{8}{2} > 2$, the analysis of unweighted cell means is of dubious worth.

Calculate: $\frac{1}{n_h} = \frac{1}{2(3)}\left(\frac{1}{8}+\frac{1}{6}+\frac{1}{5}+\frac{1}{2}+\frac{1}{8}+\frac{1}{6}\right) = 0.21389$. Thus, $n_h = 4.675$

(K) There is no output from (J)

ANALYSIS OF VARIANCE PROCEDURE

DEPENDENT VARIABLE: MY

SOURCE	DF	SUM OF SQUARES	MEAN SQUARE
MODEL	5	0.47950231	0.09590046
ERROR	0	0.00000000	0.00000000
CORRECTED TOTAL	5	0.47950231	

R-SQUARE	C.V.	STD DEV	MY MEAN
1.000000	0.0000	0.00000000	6.66027778

SOURCE	DF	ANOVA SS	F VALUE	PR > F
LOC	1	0.21596713	.	.
TYPE	2	0.13235093	.	.
TYPE*LOC	2	0.13118426	.	.

ANALYSIS OF VARIANCE PROCEDURE

CLASS LEVEL INFORMATION

CLASS	LEVELS	VALUES
TYPE	3	1 2 3
LOC	2	1 2

Type 1 = North
2 = Mesic
3 = Shrub

Loc 1 = Near
2 = Away

NUMBER OF OBSERVATIONS IN DATA SET = 6 = the number of cell means.

These are the desired SS which need to be multiplied by $n_h = 4.675$, the harmonic mean number of observations per cell.

ANOVA of Unweighted Cell Means

Source	df	SS	MS	"F"
LOC	1	1.00965	1.00965	5.19
TYPE	2	0.61874	0.30937	1.59
TYPE*LOC	2	0.61329	0.30664	1.58
ERROR	29	5.63633	0.19436	

$F^{.05}_{1,29} = 4.18$ $F^{.05}_{2,29} = 3.33$

$F^{.01}_{1,29} = 7.60$ $F^{.25}_{2,29} = 1.45$

where: LOC SS = 4.675(0.21596713) = 1.00965
TYPE SS = 4.675(0.13235093) = 0.61874
TYPE*LOC SS = 4.675(0.13118426) = 0.61329
Error SS from part (I)

These are only approximate F ratios because the cell means are given equal weight in this analysis.

K continued

ANALYSIS OF VARIANCE PROCEDURE

MEANS

LOC	N	MY
1	3	6.47055556
2	3	6.85000000

TYPE	N	MY
1	2	6.86250000
2	2	6.60833333
3	2	6.51000000

These are unweighted means, e.g. $6.471 = \frac{1}{3}(6.825 + 6.467 + 6.120)$.

Thus, they correspond to the LSMEANS from the model which includes LOC, TYPE and LOC*TYPE.

Note: The N are not appropriate on this page.

TYPE	LOC	N	MY
1	1	1	6.82500000
1	2	1	6.90000000
2	1	1	6.46666667
2	2	1	6.75000000
3	1	1	6.12000000
3	2	1	6.90000000

These are the six cell means -- compare with part A.

— They are in different order because we list TYPE LOC here and LOC TYPE in A.

The standard error of any location mean is $\sqrt{\frac{s^2}{3(n_h)}} = \sqrt{\frac{0.19436}{3(4.675)}} = 0.1177$

The standard error of any type mean is $\sqrt{\frac{0.19436}{2(4.675)}} = 0.1442$

See p. 419 of Snedecor and Cochran (7 ed.) for a discussion of calculating the correct standard errors of comparisons among row means, column means and individual cell means.

```
DATA SOYBEAN;
  TITLE SOYBEAN DATA;
  INPUT LIGHT $ HEIGHT YIELD @@;
    DEV=HEIGHT;
    X1 = (LIGHT='C');
    X2 = (LIGHT='L');
    X3 = 1 - X1 - X2;
  CARDS;
C 48 12.2  C 52 12.4  C 42 11.9  C 35 11.3  C 40 11.8  C 48 12.1  C 60 13.1
C 61 12.7  C 50 12.4  C 33 11.4  C 48 12.3  C 51 12.2  C 56 12.6  C 65 13.2
C 51 12.3  L 63 16.6  L 50 15.8  L 63 16.5  L 33 15.0  L 38 15.4  L 45 15.6
L 50 15.8  L 43 15.8  L 50 16.0  L 49 15.8  L 35 15.0  L 50 16.2  L 62 16.7
L 49 15.8  L 52 15.9  S 52  9.5  S 54  9.5  S 58  9.6  S 45  8.8  S 57  9.5
S 62  9.8  S 52  9.1  S 67 10.3  S 55  9.5  S 40  8.5  S 41  8.6  S 67 10.4
S 55  9.4  S 66 10.2  S 56  9.3
```

Create the data set SOYBEAN.

(X1, X2, X3:) These are "logical if" statements. If the statement inside the parenthesis is true, then the parenthesis has a value of 1; if it is false it has a value of zero.

```
(A) PROC STANDARD MEAN=0;
      VAR DEV;
```

— Sets mean of DEV = 0. It is equivalent to Height - $\overline{\text{Height}}$.

```
(B) PROC PRINT;
      VAR X1 X2 X3 DEV HEIGHT YIELD;
```

Prints the $\underset{\sim}{X}$ matrix plus height and Y (Yield) for (C).

```
(C) PROC REG;
      MODEL YIELD=X1 X2 X3 DEV/NOINT SEQB SS1 SS2;
      OUTPUT OUT=TRY P=YHAT R=RESID;
    C_VS_TRT:  TEST X1-.5*X2-.5*X3=0;
    L_VS_S  :  TEST X2-X3=0;
```

Model fitting the covariate deviations last.

Use TEST statements with PROC REG to test contrasts. Instead of specifying the coefficients as with ESTIMATE, use the equation that represents the contrasts under the null hypothesis.

```
(D) PROC PLOT;
      PLOT RESID*YHAT=LIGHT/VREF=0;
      PLOT YIELD*HEIGHT=LIGHT;
```

Residual plot for (C).
Plot of the response variable vs. the covariate.

```
(E) PROC GLM;
      CLASS LIGHT;
      MODEL YIELD=LIGHT HEIGHT LIGHT*HEIGHT;
```

Model fitting separate slopes after a common slope.

```
(F) PROC GLM;
      CLASS LIGHT;
      MODEL YIELD=LIGHT HEIGHT;
    LSMEANS LIGHT/STDERR;   <--- Adjusted treatment means.
    MEANS   LIGHT;  <--- Unadjusted treatment means AND means of the covariate (Height) in each level of light.
    ESTIMATE 'CONTROL VS TMT' HEIGHT 0 LIGHT 2 -1 -1/DIVISOR=2;
    ESTIMATE 'LIGHT VS SHADE' LIGHT 0 1 -1;
    ESTIMATE 'COMMON SLOPE'   HEIGHT 1;
```

SOYBEAN DATA

OBS	X1	X2	X3	DEV	HEIGHT	YIELD
1	1	0	0	-3.2	48	12.2
2	1	0	0	0.8	52	12.4
3	1	0	0	-9.2	42	11.9
4	1	0	0	-16.2	35	11.3
5	1	0	0	-11.2	40	11.8
6	1	0	0	-3.2	48	12.1
7	1	0	0	8.8	60	13.1
8	1	0	0	9.8	61	12.7
9	1	0	0	-1.2	50	12.4
10	1	0	0	-18.2	33	11.4
11	1	0	0	-3.2	48	12.3
12	1	0	0	-0.2	51	12.2
13	1	0	0	4.8	56	12.6
14	1	0	0	13.8	65	13.2
15	1	0	0	-0.2	51	12.3
16	0	1	0	11.8	63	16.6
17	0	1	0	-1.2	50	15.8
18	0	1	0	11.8	63	16.5
19	0	1	0	-18.2	33	15.0
20	0	1	0	-13.2	38	15.4
21	0	1	0	-6.2	45	15.6
22	0	1	0	-1.2	50	15.8
23	0	1	0	-3.2	48	15.8
24	0	1	0	-1.2	50	16.0
25	0	1	0	-2.2	49	15.8
26	0	1	0	-16.2	35	15.0
27	0	1	0	-1.2	50	16.2
28	0	1	0	10.8	62	16.7
29	0	1	0	-2.2	49	15.8
30	0	1	0	0.8	52	15.9
31	0	0	1	0.8	52	9.5
32	0	0	1	2.8	54	9.5
33	0	0	1	6.8	58	9.6
34	0	0	1	-6.2	45	8.8
35	0	0	1	5.8	57	9.5
36	0	0	1	10.8	62	9.8
37	0	0	1	0.8	52	9.1
38	0	0	1	15.8	67	10.3
39	0	0	1	3.8	55	9.5
40	0	0	1	-11.2	40	8.5
41	0	0	1	-10.2	41	8.6
42	0	0	1	15.8	67	10.4
43	0	0	1	3.8	55	9.4
44	0	0	1	14.8	66	10.2
45	0	0	1	4.8	56	9.3

X1: Control treatment INDicator

X2: Light treatment INDicator

X3: Shade treatment INDicator

DEV: Height - $\overline{\text{Height}}$ (deviation of the covariate from its mean)

(B) The $\underset{\sim}{X}$ matrix plus Height and Yield for part (C).

GENERAL MEANS MODEL

(C) YIELD = Treatment indicators | covariate deviations

SEQUENTIAL PARAMETER ESTIMATES

X1	12.26 $=\bar{y}_C$			
X2	12.26	15.86 $=\bar{y}_L$		
X3	12.26	15.86	9.46667 $=\bar{y}_S$	
DEV	12.369	15.9806	9.23709	.0583682 = common slope ← The first three partials are the adjusted treatment means (see p. 237).

DEP VARIABLE: YIELD

SOURCE	DF	SUM OF SQUARES	MEAN SQUARE	F VALUE	PROB>F
MODEL	4	7383.377	1845.844	110755.812	0.0001
ERROR	41	0.683301	0.016666		
U TOTAL	45	7384.060			

ROOT MSE	0.129096	R-SQUARE	~~0.9999~~
DEP MEAN	12.528889	ADJ R-SQ	~~0.9999~~
C.V.	1.03039		

NOTE: NO INTERCEPT TERM IS USED. R-SQUARE IS REDEFINED.

VARIABLE	DF	PARAMETER ESTIMATE	STANDARD ERROR	T FOR H0: PARAMETER=0	PROB > |T|	TYPE I SS	
X1	1	12.368954	0.033592	368.213	0.0001	2254.614	$\bar{y}_C$ (adj) = 12.37
X2	1	15.980628	0.033650	474.906	0.0001	3773.094	$\bar{y}_L$ (adj) = 15.98
X3	1	9.237085	0.034469	267.984	0.0001	1344.267	$\bar{y}_S$ (adj) = 9.24
DEV	1	slope=0.058368	0.002231513	26.156	0.0001	11.402032	

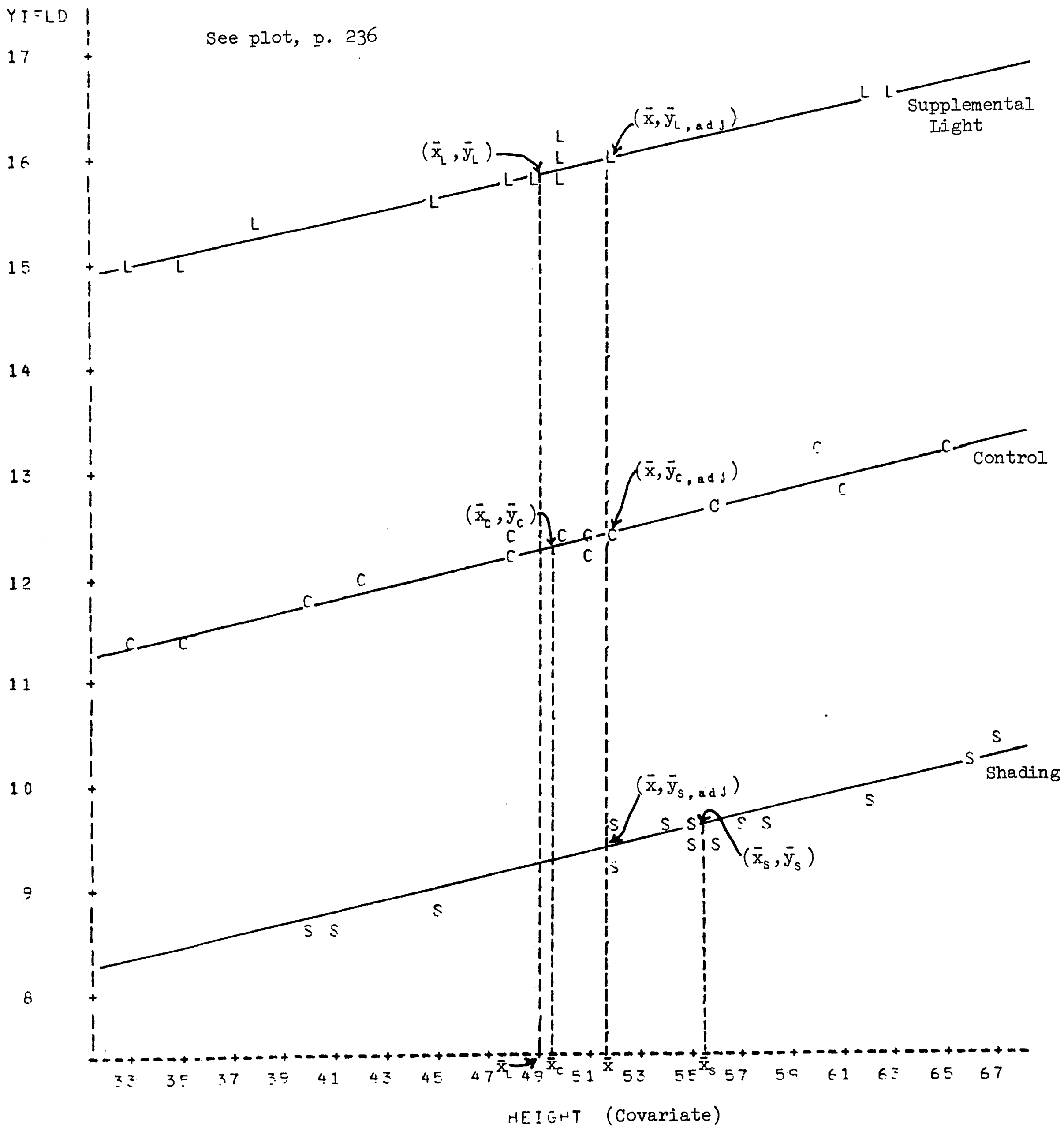
VARIABLE	DF	TYPE II SS
X1	1	2259.578
X2	1	3758.754
X3	1	1196.866
DEV	1	11.402032

TEST: C_VS_TRT	NUMERATOR:	0.562358	DF: 1	F VALUE:	33.7431
	DENOMINATOR:	.0166659	DF: 41	PROB >F :	0.0001
TEST: L_VS_S	NUMERATOR:	315.604	DF: 1	F VALUE:	8937.1324
	DENOMINATOR:	.0166659	DF: 41	PROB >F :	0.0001

TESTS of contrasts

These contrasts test differences between adjusted means.
The ESTIMATE option in GLM would provide the estimate and standard error (see (G)).

D PLOT OF YIELD*HEIGHT SYMBOL IS VALUE OF LIGHT



NOTE: 5 OBS HIDDEN

SOYBEAN DATA

GENERAL LINEAR MODELS PROCEDURE

Ⓔ Treatments/common slope/separate slopes model

CLASS LEVEL INFORMATION

CLASS	LEVELS	VALUES
LIGHT	3	C L S

NUMBER OF OBSERVATIONS IN DATA SET = 45

SOYBEAN DATA

GENERAL LINEAR MODELS PROCEDURE

DEPENDENT VARIABLE: YIELD

SOURCE	DF	SUM OF SQUARES	MEAN SQUARE	F VALUE
MODEL	5	319.66578021	63.93315604	4110.01
ERROR	39	0.60666423	0.01555549	PR > F
CORRECTED TOTAL	44	320.27244444		0.0001

R-SQUARE	C.V.	STD DEV	YIELD MEAN
0.998106	0.9955	0.12472166	12.52888889

SOURCE	DF	TYPE I SS	F VALUE	PR > F
LIGHT	2	308.18711111	9906.05	0.0001
HEIGHT	1	11.40203184	732.99	0.0001
HEIGHT*LIGHT	2	0.07663726	2.46	0.0983

this line tests $H_0 : \beta_1 = \beta_2 = \beta_3$ $(=\beta)$ vs. H_a : different slopes needed.

SOURCE	DF	TYPE IV SS	F VALUE	PR > F
LIGHT	2	10.53027719	338.47	0.0001
HEIGHT	1	11.47084949	737.41	0.0001
HEIGHT*LIGHT	2	0.07663726	2.46	0.0983

SOYBEAN DATA

GENERAL LINEAR MODELS PROCEDURE

Ⓕ Treatments/common slope model

DEPENDENT VARIABLE: YIELD

SOURCE	DF	SUM OF SQUARES	MEAN SQUARE	F VALUE
MODEL	3	319.58914296	106.52971432	6392.06
ERROR	41	0.68330149	0.01666589	PR > F
CORRECTED TOTAL	44	320.27244444		0.0001

R-SQUARE	C.V.	STD DEV	YIELD MEAN
0.997866	1.0304	0.12909644	12.52888889

SOURCE	DF	TYPE I SS	F VALUE	PR > F
LIGHT $= R(L\|\mu)$	2	308.18711111	9246.04	0.0001
HEIGHT $= R(\beta\|L,\mu)$	1	11.40203184	684.15	0.0001

$$R(\beta|\mu) = R(L,\beta|\mu) - R(L|\beta,\mu)$$
$$= 319.58914 - 317.81096$$
$= 1.77818$ = SS due to fitting a common slope before the treatments.

SOURCE	DF	TYPE IV SS	F VALUE	PR > F
LIGHT $= R(L\|\beta,\mu)$	2	317.81096489	9534.77	0.0001
HEIGHT	1	11.40203184	684.15	0.0001

PARAMETER	ESTIMATE	T FOR H0: PARAMETER=0	PR > |T|	STD ERROR OF ESTIMATE
CONTROL VS TMT (1)	-0.23990239	-5.81	0.0001	0.04129927
LIGHT VS SHADE (2)	6.74354249	137.61	0.0001	0.04900394
COMMON SLOPE (3)	0.05836819	26.16	0.0001	0.00223151

(1) and (2) are the contrasts among adjusted treatment means (same as the contrasts used in the TESTs of part Ⓒ).

(3) is the estimate of the common slope of the covariate.

Note that the F-value output for (2) in PROC REG is not correct.

MEANS = unadjusted treatment means

LIGHT	N	YIELD	HEIGHT
C	15	12.2600000	49.3333333
L	15	15.8600000	49.1333333
S	15	9.4666667	55.1333333

LEAST SQUARES MEANS = adjusted treatment means and their standard errors.

LIGHT	YIELD LSMEAN	STD ERR LSMEAN	PROB > |T| H0:LSMEAN=0
C	12.3089540	0.0335918	0.0001
L	15.9806276	0.0336501	0.0001
S	9.2370851	0.0344688	0.0001

Compare the MEANS and LSMEANS with the SEQB output of PROC REG in Ⓒ.

POTATO SCAB DATA - Comparison of regression lines in a 2 X 3 factorial experiment (two qualitative levels x three quantitative levels). P. 97, Cochran and Cox, 1957.

```
DATA SCAB;
INPUT YIELD TMT $ LEVEL X0 X01 X02 X1 X11 X12 X2 X21 X22;
CARDS;
```

A

```
PROC PRINT N;
```

B

```
PROC REG;
     MODEL  YIELD = X0 X01 X02 X1 X11 X12 X2 X21 X22 /
                    NOINT P SEQB SS1 CLM;
```

B and C correspond to fitting a sequence of regression models

C

```
PROC REG;
     MODEL  YIELD = X01 X02 X1 X2 / NOINT P SEQB SS1  CLM;
     OUTPUT OUT=NEW P=YHAT R=RES U95M=UPPER L95M=LOWER;
```

D

```
PROC PLOT DATA=NEW;
     PLOT YIELD*LEVEL=TMT;
     PLOT RES*YHAT / VREF=0;
     PLOT YHAT*LEVEL='P' UPPER*LEVEL='U' LOWER*LEVEL='L' / OVERLAY;
```

A plot of the raw data as well as a residual plot and plot of predicted values for the model in part C.

E

```
PROC UNIVARIATE NORMAL PLOT DATA=NEW; VAR RES;
```

More analysis of residuals from the model in part C.

F

```
PROC GLM DATA=SCAB;  CLASS TMT LEVEL;
     MODEL  YIELD = TMT*LEVEL / NOINT P;
     ESTIMATE 'TMT'      TMT*LEVEL 1 1 1 -1 -1 -1   / DIVISOR=3;
     ESTIMATE 'B LIN'    TMT*LEVEL -4 -1 5 -4 -1 5 / DIVISOR=84;
     ESTIMATE 'T*B LIN'  TMT*LEVEL -4 -1 5 4 1 -5  / DIVISOR=42;
     ESTIMATE 'B QUAD'   TMT*LEVEL 2 -3 1 2 -3 1   / DIVISOR=108;
     ESTIMATE 'T*B QUAD' TMT*LEVEL 2 -3 1 -2 3 -1  / DIVISOR=54;
```

F Fitting the cell means model and examining single degree-of-freedom contrasts which correspond to linear, quadratic, treatment and the respective interaction terms. The results are identical to those of part B.

Recall that $b_{2.01} = \Sigma \ell_i y_i$ where the ℓ_i's may be computed from, say, the ORTHO algorithm. In this example $\ell_1 = \frac{2}{54}$, $\ell_2 = \frac{-3}{54}$ and $\ell_3 = \frac{1}{54}$. If the levels of the quantitative factor had been equally spaced, then the ℓ_i's could have been obtained from a table of orthogonal polynomial coefficients.

(A) The data and indicator variables

OBS	YIELD	TMT	LEVEL	X0	X01	X02	X1	X11	X12	X2	X21	X22
1	9	F	3	1	1	0	3	3	0	9	9	0
2	9	F	3	1	1	0	3	3	0	9	9	0
3	16	F	3	1	1	0	3	3	0	9	9	0
4	4	F	3	1	1	0	3	3	0	9	9	0
5	30	S	3	1	0	1	3	0	3	9	0	9
6	7	S	3	1	0	1	3	0	3	9	0	9
7	21	S	3	1	0	1	3	0	3	9	0	9
8	9	S	3	1	0	1	3	0	3	9	0	9
9	16	F	6	1	1	0	6	6	0	36	36	0
10	10	F	6	1	1	0	6	6	0	36	36	0
11	18	F	6	1	1	0	6	6	0	36	36	0
12	18	F	6	1	1	0	6	6	0	36	36	0
13	18	S	6	1	0	1	6	0	6	36	0	36
14	24	S	6	1	0	1	6	0	6	36	0	36
15	12	S	6	1	0	1	6	0	6	36	0	36
16	19	S	6	1	0	1	6	0	6	36	0	36
17	10	F	12	1	1	0	12	12	0	144	144	0
18	4	F	12	1	1	0	12	12	0	144	144	0
19	4	F	12	1	1	0	12	12	0	144	144	0
20	5	F	12	1	1	0	12	12	0	144	144	0
21	17	S	12	1	0	1	12	0	12	144	0	144
22	7	S	12	1	0	1	12	0	12	144	0	144
23	16	S	12	1	0	1	12	0	12	144	0	144
24	17	S	12	1	0	1	12	0	12	144	0	144

N=24

B Fitting the model sequence X0|X01 X02|X1|X11 X12|X2|X21 X22

which is: common mean|separate means|common linear|separate linear|common quadratic|separate quadratic

SEQUENTIAL PARAMETER ESTIMATES

X0	13.3333 = $\bar{y}$								
X01	16.4167	-6.16667 = $\bar{y}_F - \bar{y}_S$							
X02	16.4167	-6.16667	1.4E-14 = 0						
X1	19.6458	-6.16667	1.7E-14	-0.46131 = $b_{1.0}$ (common linear)					
X11	18.75	-4.375	1.7E-14	-.333333	-.255952 = $b_{F1.0} - b_{S1.0}$				
X12	18.75	-4.375	1.7E-14	-.333333	-.255952	1.1E-13 = 0			
X2	6.77083	-4.375	1.8E-14	3.77381	-.255952	1.6E-13	-.266204 = $b_{2.01}$ (common quadratic)		
X21	12.9167	-16.6667	3.3E-14	1.66667	3.95833	-2.2E-13	-0.12963	-.273148 = $b_{F2.01} - b_{S2.01}$	
X22	12.9167	-16.6667	3.3E-14	1.66667	3.95833	-2.2E-13	-0.12963	-.273148	1.2E-12 = 0

DEP VARIABLE: YIELD

SOURCE	DF	SUM OF SQUARES	MEAN SQUARE	F VALUE	PROB>F
MODEL	6	4721.000	786.833	22.374	0.0001
ERROR	18	633.000	35.166667		
U TOTAL	24	5354.000			

Standard errors and hypothesis tests are most easily determined from part F.

ROOT MSE	5.930149	~~R-SQUARE~~	~~0.8818~~
DEP MEAN	13.333333	~~ADJ R-SQ~~	~~0.8489~~
C.V.	44.47612		

NOTE: NO INTERCEPT TERM IS USED. R-SQUARE IS REDEFINED.

NOTE: MODEL IS NOT FULL RANK. LEAST SQUARES SOLUTIONS FOR THE PARAMETERS ARE NOT UNIQUE. SOME STATISTICS WILL BE MISLEADING. A REPORTED DF OF 0 OR B MEANS THAT THE ESTIMATE IS BIASED. THE FOLLOWING PARAMETERS HAVE BEEN SET TO 0, SINCE THE VARIABLES ARE A LINEAR COMBINATION OF OTHER VARIABLES AS SHOWN.

X02	=+X0	-1*X01
X12	=+X1	-1*X11
X22	=+X2	-1*X21

not of use.

VARIABLE	DF	PARAMETER ESTIMATE	STANDARD ERROR	T FOR H0: PARAMETER=0	PROB > |T|	TYPE I SS
X0	B	12.916667	9.932877	1.300	0.2099	4266.667
X01	B	-16.666667	14.047209	-1.186	0.2509	228.167
X02	0	0	.	.	.	0
X1	B	1.666667	3.202646	0.520	0.6091	71.502976
X11	B	3.958333	4.529226	0.874	0.3937	5.502976
X12	0	0	.	.	.	0
X2	B	-0.129630	0.205450	-0.631	0.5360	118.080
X21	B	-0.273148	0.290550	-0.940	0.3596	31.080357

B Results of the P and CIM option

OBS	ACTUAL	PREDICT VALUE (Cell Means)	STD ERR PREDICT	LOWER95% MEAN	UPPER95% MEAN	RESIDUAL
1	9.000	9.500	2.965	3.271	15.729	-.500000
2	9.000	9.500	2.965	3.271	15.729	-.500000
3	16.000	9.500	2.965	3.271	15.729	6.500
4	4.000	9.500	2.965	3.271	15.729	-5.500
5	30.000	16.750	2.965	10.521	22.979	13.250
6	7.000	16.750	2.965	10.521	22.979	-9.750
7	21.000	16.750	2.965	10.521	22.979	4.250
8	9.000	16.750	2.965	10.521	22.979	-7.750
9	16.000	15.500	2.965	9.271	21.729	0.500000
10	10.000	15.500	2.965	9.271	21.729	-5.500
11	18.000	15.500	2.965	9.271	21.729	2.500
12	18.000	15.500	2.965	9.271	21.729	2.500
13	18.000	18.250	2.965	12.021	24.479	-.250000
14	24.000	18.250	2.965	12.021	24.479	5.750
15	12.000	18.250	2.965	12.021	24.479	-6.250
16	19.000	18.250	2.965	12.021	24.479	0.750000
17	10.000	5.750	2.965	-.479346	11.979	4.250
18	4.000	5.750	2.965	-.479346	11.979	-1.750
19	4.000	5.750	2.965	-.479346	11.979	-1.750
20	5.000	5.750	2.965	-.479346	11.979	-.750000
21	17.000	14.250	2.965	8.021	20.479	2.750
22	7.000	14.250	2.965	8.021	20.479	-7.250
23	16.000	14.250	2.965	8.021	20.479	1.750
24	17.000	14.250	2.965	8.021	20.479	2.750

SUM OF RESIDUALS 2.34479E-13
SUM OF SQUARED RESIDUALS 633

Ⓒ Fitting the model sequence X01 X02|X1|X2

which is: separate means|common linear|common quadratic

SEQUENTIAL PARAMETER ESTIMATES

X01	10.25 = $\bar{y}_F$			
X02	10.25	16.4167 = $\bar{y}_S$		
X1	13.4792	19.6458	-0.46131 = $b_{1.0}$	
X2	1.5	7.66667	3.64583	-.266204 = $b_{2.01}$

DEP VARIABLE: YIELD

SOURCE	DF	SUM OF SQUARES	MEAN SQUARE	F VALUE	PROB>F
MODEL	4	4684.417	1171.104	34.980	0.0001
ERROR	20	669.583	33.479167		
U TOTAL	24	5354.000			

ROOT MSE	5.786118	~~R-SQUARE~~	~~0.8749~~
DEP MEAN	13.333333	~~ADJ R-SQ~~	~~0.8562~~
C.V.	43.39589		

NOTE: NO INTERCEPT TERM IS USED. R-SQUARE IS REDEFINED.

VARIABLE	DF	PARAMETER ESTIMATE	STANDARD ERROR	T FOR H0: PARAMETER=0	PROB > |T|	TYPE I SS
X01	1	1.500000	6.954049	0.216	0.8314	1260.750
X02	1	7.666667	6.954049	1.102	0.2833	3234.083
X1	1	3.645833	2.209610	1.650	0.1146	71.502976
X2	1	-0.266204	0.141747	-1.878	0.0750	118.080

not useful

OBS	ACTUAL	PREDICT VALUE (=restricted cell means)	STD ERR PREDICT	LOWER95% MEAN	UPPER95% MEAN	RESIDUAL
1	9.000	10.042	2.362	5.114	14.969	-1.042
2	9.000	10.042	2.362	5.114	14.969	-1.042
3	16.000	10.042	2.362	5.114	14.969	5.958
4	4.000	10.042	2.362	5.114	14.969	-6.042
5	30.000	16.208	2.362	11.281	21.136	13.792
6	7.000	16.208	2.362	11.281	21.136	-9.208
7	21.000	16.208	2.362	11.281	21.136	4.792
8	9.000	16.208	2.362	11.281	21.136	-7.208
9	16.000	13.792	2.362	8.864	18.719	2.208
10	10.000	13.792	2.362	8.864	18.719	-3.792
11	18.000	13.792	2.362	8.864	18.719	4.208
12	18.000	13.792	2.362	8.864	18.719	4.208
13	18.000	19.958	2.362	15.031	24.886	-1.958
14	24.000	19.958	2.362	15.031	24.886	4.042
15	12.000	19.958	2.362	15.031	24.886	-7.958
16	19.000	19.958	2.362	15.031	24.886	-.958333
17	10.000	6.917	2.362	1.989	11.844	3.083
18	4.000	6.917	2.362	1.989	11.844	-2.917
19	4.000	6.917	2.362	1.989	11.844	-2.917
20	5.000	6.917	2.362	1.989	11.844	-1.917
21	17.000	13.083	2.362	8.156	18.011	3.917
22	7.000	13.083	2.362	8.156	18.011	-6.083
23	16.000	13.083	2.362	8.156	18.011	2.917
24	17.000	13.083	2.362	8.156	18.011	3.917

SUM OF RESIDUALS 1.34115E-13
SUM OF SQUARED RESIDUALS 669.5833

GENERAL LINEAR MODELS PROCEDURE

DEPENDENT VARIABLE: YIELD

SOURCE	DF	SUM OF SQUARES	MEAN SQUARE	F VALUE
MODEL	6	4721.00000000	786.83333333	22.37
ERROR	18	633.00000000	35.16666667	PR > F
UNCORRECTED TOTAL	24	5354.00000000		0.0001

R-SQUARE	C.V.	STD DEV	YIELD MEAN
0.881771	44.4761	5.93014896	13.33333333

SOURCE	DF	TYPE I SS	F VALUE	PR > F
TMT*LEVEL	6	4721.00000000	22.37	0.0001

SOURCE	DF	TYPE IV SS	F VALUE	PR > F
TMT*LEVEL	6	4721.00000000	22.37	0.0001

PARAMETER	ESTIMATE	T FOR H0: PARAMETER=0	PR > |T|	STD ERROR OF ESTIMATE
TMT	-6.16666667	-2.55	0.0202	2.42097317
B LIN	-0.46130952	-1.43	0.1710	0.32351615
T*B LIN	-0.25595238	-0.40	0.6971	0.64703230
B QUAD	-0.26620370	-1.83	0.0835	0.14527499
T*B QUAD	-0.27314815	-0.94	0.3596	0.29054999

OBSERVATION	OBSERVED VALUE	PREDICTED VALUE (= cell means)	RESIDUAL
1	9.00000000	9.50000000	-0.50000000
2	9.00000000	9.50000000	-0.50000000
3	16.00000000	9.50000000	6.50000000
4	4.00000000	9.50000000	-5.50000000
5	30.00000000	16.75000000	13.25000000
6	7.00000000	16.75000000	-9.75000000
7	21.00000000	16.75000000	4.25000000
8	9.00000000	16.75000000	-7.75000000
9	16.00000000	15.50000000	0.50000000
10	10.00000000	15.50000000	-5.50000000
11	18.00000000	15.50000000	2.50000000
12	18.00000000	15.50000000	2.50000000

(F) General Means Model

GENERAL LINEAR MODELS PROCEDURE

CLASS LEVEL INFORMATION

CLASS	LEVELS	VALUES
TMT	2	F S
LEVEL	3	3 6 12

NUMBER OF OBSERVATIONS IN DATA SET = 24

Note that the parameter estimates are the same as those given in part (B) under the SEQB output. Here we also have the standard errors and t-tests; however, we had to calculate the ℓ_i's for each contrast.

(Cell means)

TMT \ LEVEL	3	6	12
F	9.50	15.50	5.75
S	16.75	18.25	14.25

The standard error of each cell mean is $\sqrt{\frac{35.1667}{4}} = 2.965$. The cell means and standard errors could have been printed by using the option:

LSMEANS TMT*LEVEL/STDERR;

F continued GENERAL LINEAR MODELS PROCEDURE

DEPENDENT VARIABLE: YIELD

OBSERVATION	OBSERVED VALUE	PREDICTED VALUE	RESIDUAL
13	18.00000000	18.25000000	-0.25000000
14	24.00000000	18.25000000	5.75000000
15	12.00000000	18.25000000	-6.25000000
16	19.00000000	18.25000000	0.75000000
17	10.00000000	5.75000000	4.25000000
18	4.00000000	5.75000000	-1.75000000
19	4.00000000	5.75000000	-1.75000000
20	5.00000000	5.75000000	-0.75000000
21	17.00000000	14.25000000	2.75000000
22	7.00000000	14.25000000	-7.25000000
23	16.00000000	14.25000000	1.75000000
24	17.00000000	14.25000000	2.75000000

SUM OF RESIDUALS	0.00000000
SUM OF SQUARED RESIDUALS	633.00000000
SUM OF SQUARED RESIDUALS - ERROR SS	0.00000000
FIRST ORDER AUTOCORRELATION	-0.63467615
DURBIN-WATSON D	3.25701027

Note: In order to get the classical ANOVA table as well as the single degree-of-freedom contrasts and means with standard errors we could have used the statements:

```
PROC GLM; CLASS TMT LEVEL;
     MODEL YIELD = TMT LEVEL TMT*LEVEL/P CLM;
    [ same ESTIMATE statements]
     LSMEANS TMT LEVEL TMT*LEVEL/STDERR;
```

Note: To obtain the same results as in C we would fit the model with interaction restricted to be zero That is,

```
PROC GLM; CLASS TMT LEVEL;
     MODEL YIELD = TMT LEVEL /P CLM;
```

ALCOHOL-DRUG DATA, p. 280.

Analysis of a Split-Unit Experiment

```
DATA WHOLE;
INPUT Y1 Y2 Y3;
SUBJECT = _N_;
ALCOHOL = 'YES';
IF _N_ > 6 THEN ALCOHOL='NO';
YS = (Y1+Y2+Y3)/SQRT(3);
XO = 1;
CARDS;
```

Y1 = Drug A response

Y2 = Drug B response

Y3 = Control response

$$YS = \frac{Y_1+Y_2+Y_3}{\sqrt{3}}$$

```
PROC SORT; BY ALCOHOL;
```

Ⓐ
```
PROC PRINT N; BY ALCOHOL;
```

Ⓑ
```
PROC GLM; CLASS ALCOHOL;
     MODEL  YS = XO ALCOHOL / NOINT;
     LSMEANS ALCOHOL / STDERR;
     ESTIMATE 'DIFFERENCE' ALCOHOL 1 -1;
```

Ⓒ
```
DATA SPLIT; SET WHOLE;
     Y=Y1; DRUG='A'; OUTPUT;
     Y=Y2; DRUG='B'; OUTPUT;
     Y=Y3; DRUG='C'; OUTPUT;
     DROP Y1-Y3 YS;
```

Rearranging the data to enable analysis of the subplot factor.

Ⓓ
```
PROC PRINT N;

PROC SORT; BY ALCOHOL SUBJECT;
```

PROC SORT must be used on the CLASS variables used in the ABSORB statement below. ABSORBing the whole plot factors reduces the storage requirements and hence the time and cost of the analysis.

Ⓔ
```
PROC GLM;
     ABSORB ALCOHOL SUBJECT;
     CLASS DRUG ALCOHOL;
     MODEL  Y = DRUG ALCOHOL*DRUG;
     ESTIMATE 'MAIN EFFECT: AB VS C' DRUG -1 -1 2 / DIVISOR=2;
     ESTIMATE 'MAIN EFFECT: A VS B ' DRUG  1 -1 0;
```

Effects model for the subplot factor and main effects contrasts.

Ⓕ
```
PROC GLM;
     ABSORB ALCOHOL SUBJECT;
```

```
CLASSES ALCOHOL DRUG;
MODEL  Y = ALCOHOL*DRUG / NOINT SS1 SS2;
ESTIMATE 'AB VS CONTROL @ NO'
         ALCOHOL*DRUG -1 -1 2 0 0 0 / DIVISOR=2 E;
ESTIMATE 'A VS B @ NO ALCOHOL'
         ALCOHOL*DRUG 1 -1 0 0 0 0;
ESTIMATE 'AB VS CONTROL @ YES'
         ALCOHOL*DRUG 0 0 0 -1 -1 2 / DIVISOR=2;
ESTIMATE 'A VS B @ YES ALCOHOL'
         ALCOHOL*DRUG 0 0 0 1 -1 0;
```

General means model and simple effects contrasts.

(G)

```
PROC GLM; CLASSES SUBJECT ALCOHOL DRUG;
      MODEL  Y = XO ALCOHOL SUBJECT(ALCOHOL)
                 DRUG ALCOHOL*DRUG / SS1 SS2 NOINT;
      TEST H=ALCOHOL E=SUBJECT(ALCOHOL) / HTYPE=1 ETYPE=1;
      CONTRAST 'ALCOHOL DIFFERENCE' ALCOHOL 1 -1 /
               E=SUBJECT(ALCOHOL) ETYPE=1;
      ESTIMATE 'AB VS CONTROL @ NO' DRUG -1 -1 2
               ALCOHOL*DRUG -1 -1 2 0 0 0 / DIVISOR=2 E;
      ESTIMATE 'A VS B @ NO ALCOHOL' DRUG 1 -1 0
               ALCOHOL*DRUG  1 -1 0 0 0 0;
      ESTIMATE 'AB VS CONTROL @ YES' DRUG -1 -1 2
               ALCOHOL*DRUG 0 0 0 -1 -1 2 / DIVISOR=2;
      ESTIMATE 'A VS B @ YES ALCOHOL' DRUG 1 -1 0
               ALCOHOL*DRUG 0 0 0 1 -1 0;
      OUTPUT OUT=NEW2 P=P R=R;
```

This analysis performs both the whole-plot and sub-plot analyses all at one time. The simple effects contrasts are more difficult to specify and the computing costs are much steeper.

(H)

```
PROC PLOT; PLOT R*P='*' / VREF=0;
```

Analysis of residuals

(I)

```
PROC ANOVA; CLASSES SUBJECT ALCOHOL DRUG;
       MODEL  Y = ALCOHOL SUBJECT(ALCOHOL) DRUG ALCOHOL*DRUG;
       TEST H=ALCOHOL E=SUBJECT(ALCOHOL);
       MEANS ALCOHOL SUBJECT(ALCOHOL) DRUG ALCOHOL*DRUG / DEPONLY;
```

PROC ANOVA may be used since this experiment is balanced. The ESTIMATE option is not available, but the cell means and SS for the ANOVA table are computed and F-tests made.

---------------------------------- ALCOHOL=NO ----------------------------------

OBS	Y1	Y2	Y3	SUBJECT	YS	X0
1	2.83	2.55	2.63	7	4.62458	1
2	2.93	2.42	2.73	8	4.66499	1
3	3.58	3.99	3.38	9	6.32199	1
4	2.98	3.07	2.78	10	5.09800	1
5	2.32	2.15	2.12	11	3.80474	1
6	2.73	3.23	2.53	12	4.90170	1

N=6

---------------------------------- ALCOHOL=YES ----------------------------------

OBS	Y1	Y2	Y3	SUBJECT	YS	X0
7	3.56	4.04	3.26	1	6.27002	1
8	3.79	3.88	3.49	2	6.44323	1
9	4.09	5.32	3.79	3	7.62102	1
10	3.10	4.38	2.80	4	5.93516	1
11	3.33	3.63	3.03	5	5.76773	1
12	3.35	3.63	3.05	6	5.79082	1

N=6

(A) See p. 283 of Allen and Cady for a discussion of the assumed model.

GENERAL LINEAR MODELS PROCEDURE

CLASS LEVEL INFORMATION

CLASS	LEVELS	VALUES
ALCOHOL	2	NO YES

NUMBER OF OBSERVATIONS IN DATA SET = 12

(E) Analysis of sums to test for whole-plot (alcohol) differences.

DEPENDENT VARIABLE: YS

Compare SS found on this page with those in, say, analysis (F).

SOURCE	DF	SUM OF SQUARES	MEAN SQUARE	F VALUE
MODEL	2	382.70960556	191.35480278	328.00
ERROR	10	5.83396111	0.58339611	PR > F
UNCORRECTED TOTAL	12	388.54356667		0.0001

= SS due to SUBJECT(ALCOHOL).

R-SQUARE	C.V.	STD DEV	YS MEAN
0.984935	13.6304	0.76380371	5.60366549

SOURCE	DF	TYPE I SS	F VALUE	PR > F
XO	1	376.81280278	645.90	0.0001
ALCOHOL	1	5.89680278	10.11	0.0098

SOURCE	DF	TYPE IV SS	F VALUE	PR > F
XO	0	0.00000000	.	.
ALCOHOL	1	5.89680278	10.11	0.0098

PARAMETER	ESTIMATE	T FOR H0: PARAMETER=0	PR > |T|	STD ERROR OF ESTIMATE
DIFFERENCE	-1.40199890	-3.18	0.0098	0.44098228

LEAST SQUARES MEANS

ALCOHOL	YS LSMEAN	STD ERR LSMEAN	PROB > |T| H0:LSMEAN=0
NO	4.90266604	0.31182156	0.0001
YES	6.30466494	0.31182156	0.0001

(D)

OBS	SUBJECT	ALCOHOL	XO	Y	DRUG
1	7	NO	1	2.83	A
2	7	NO	1	2.55	B
3	7	NO	1	2.63	C
4	8	NO	1	2.93	A
5	8	NO	1	2.42	B
6	8	NO	1	2.73	C
7	9	NO	1	3.58	A
8	9	NO	1	3.99	B
9	9	NO	1	3.38	C
10	10	NO	1	2.98	A
11	10	NO	1	3.07	B
12	10	NO	1	2.78	C
13	11	NO	1	2.32	A
14	11	NO	1	2.15	B
15	11	NO	1	2.12	C
16	12	NO	1	2.73	A
17	12	NO	1	3.23	B
18	12	NO	1	2.53	C
19	1	YES	1	3.56	A
20	1	YES	1	4.04	B
21	1	YES	1	3.26	C
22	2	YES	1	3.79	A
23	2	YES	1	3.88	B
24	2	YES	1	3.49	C
25	3	YES	1	4.09	A
26	3	YES	1	5.32	B
27	3	YES	1	3.79	C
28	4	YES	1	3.10	A
29	4	YES	1	4.38	B
30	4	YES	1	2.80	C
31	5	YES	1	3.33	A
32	5	YES	1	3.63	B
33	5	YES	1	3.03	C
34	6	YES	1	3.35	A
35	6	YES	1	3.63	B
36	6	YES	1	3.05	C

GENERAL LINEAR MODELS PROCEDURE

CLASS LEVEL INFORMATION

CLASS	LEVELS	VALUES
DRUG	3	A B C
ALCOHOL	2	NO YES

NUMBER OF OBSERVATIONS IN DATA SET = 36

GENERAL LINEAR MODELS PROCEDURE

DEPENDENT VARIABLE: Y

SOURCE	DF	SUM OF SQUARES	MEAN SQUARE	F VALUE
MODEL	15	14.47667500	0.96511167	13.66
ERROR	20	1.41262222	0.07063111	PR > F
CORRECTED TOTAL	35	15.88929722		0.0001

R-SQUARE	C.V.	STD DEV	Y MEAN
0.911096	8.2146	0.26576514	3.23527778

SOURCE	DF	TYPE I SS	F VALUE	PR > F
ALCOHOL	1	5.89680278	~~83.49~~	~~0.0001~~
SUBJECT(ALCOHOL)	10	5.83396111	~~8.26~~	~~0.0001~~
DRUG	2	1.87722222	13.29	0.0002
DRUG*ALCOHOL	2	0.86868889	6.15	0.0083

SOURCE	DF	TYPE IV SS	F VALUE	PR > F
DRUG	2	1.87722222	13.29	0.0002
DRUG*ALCOHOL	2	0.86868889	6.15	0.0083

Note that ALCOHOL and SUBJECT(ALCOHOL) are not included in the Type IV SS.

PARAMETER	ESTIMATE	T FOR H0: PARAMETER=0	PR > |T|	STD ERROR OF ESTIMATE
MAIN EFFECT: AB VS C	-0.40416667	-4.30	0.0003	0.09396217
MAIN EFFECT: A VS B	-0.30833333	-2.84	0.0101	0.10849817

These contrasts are of little interest since the DRUG*ALCOHOL interaction is clearly important.

(F) GENERAL LINEAR MODELS PROCEDURE

General means model with whole-plot factor ABSORBed.

DEPENDENT VARIABLE: Y

SOURCE	DF	SUM OF SQUARES	MEAN SQUARE	F VALUE
MODEL	15	14.47667500	0.96511167	13.66
ERROR	20	1.41262222	0.07063111	PR > F
CORRECTED TOTAL	35	15.88929722		0.0001

R-SQUARE	C.V.	STD DEV	Y MEAN
0.911096	8.2146	0.26576514	3.23527778

SOURCE	DF	TYPE I SS	F VALUE	PR > F
ALCOHOL	1	5.89680278	83.49	0.0001
SUBJECT(ALCOHOL)	10	5.83396111	8.26	0.0001
ALCOHOL*DRUG	4	2.74591111	9.72	0.0002

SOURCE	DF	TYPE II SS	F VALUE	PR > F
ALCOHOL*DRUG	4	2.74591111	9.72	0.0002

PARAMETER	ESTIMATE	T FOR H0: PARAMETER=0	PR > |T|	STD ERROR OF ESTIMATE
AB VS CONTROL @ NO	-0.20333333	-1.53	0.1416	0.13288257
A VS B @ NO ALCOHOL	-0.00666667	-0.04	0.9658	0.15343958
AB VS CONTROL @ YES	-0.60500000	-4.55	0.0002	0.13288257
A VS B @ YES ALCOHOL	-0.61000000	-3.98	0.0007	0.15343958

This is a set of simple effects contrasts (see Table 22.3, p. 281 - Allen and Cady).

Ⓖ Combined whole-plot and split-plot analysis with simple effects contrasts.

GENERAL LINEAR MODELS PROCEDURE

DEPENDENT VARIABLE: Y

SOURCE	DF	SUM OF SQUARES	MEAN SQUARE	F VALUE
MODEL	16	391.28947778	24.45559236	346.24
ERROR	20	1.41262222	0.07063111	PR > F
UNCORRECTED TOTAL	36	392.70210000		0.0001

R-SQUARE	C.V.	STD DEV	Y MEAN
0.996403	8.2146	0.26576514	3.23527778

SOURCE	DF	TYPE I SS	F VALUE	PR > F
XO	1	376.81280278	5334.94	~~0.0001~~
ALCOHOL	1	5.89680278	83.49	~~0.0001~~
SUBJECT(ALCOHOL)	10	5.83396111	8.26	0.0001
DRUG	2	1.87722222	13.29	0.0002
ALCOHOL*DRUG	2	0.86868889	6.15	0.0083

SOURCE	DF	TYPE II SS	F VALUE	PR > F
XO	0	0.00000000	.	.
ALCOHOL	1	5.89680278	83.49	~~0.0001~~
SUBJECT(ALCOHOL)	10	5.83396111	8.26	0.0001
DRUG	2	1.87722222	13.29	0.0002
ALCOHOL*DRUG	2	0.86868889	6.15	0.0083

TESTS OF HYPOTHESES USING THE TYPE I MS FOR SUBJECT(ALCOHOL) AS AN ERROR TERM

SOURCE	DF	TYPE I SS	F VALUE	PR > F
ALCOHOL	1	5.89680278	10.11	0.0098

TESTS OF HYPOTHESES USING THE TYPE I MS FOR SUBJECT(ALCOHOL) AS AN ERROR TERM

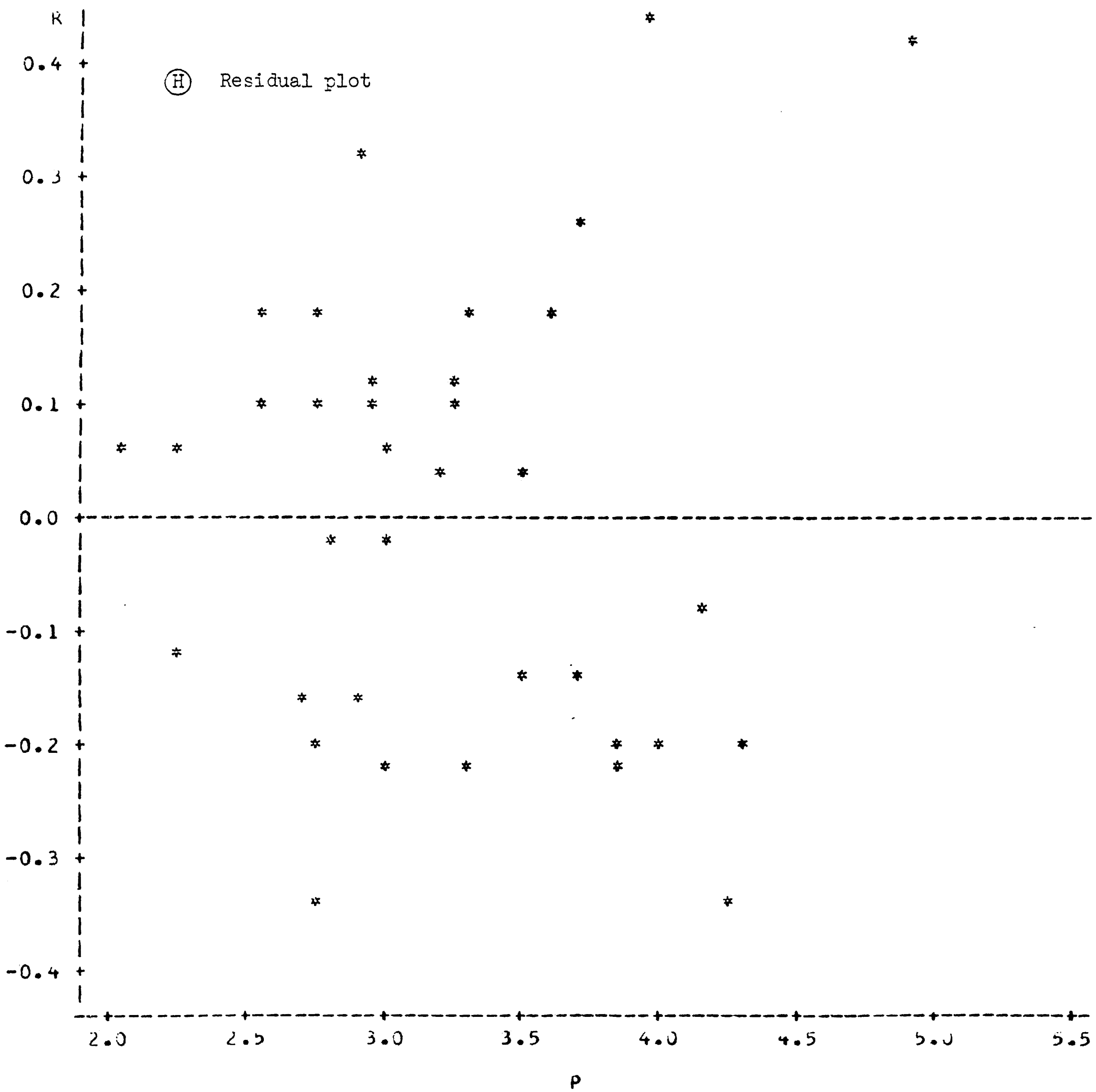
CONTRAST	DF	SS	F VALUE	PR > F
ALCOHOL DIFFERENCE	1	5.89680278	10.11	0.0098

Compare with the analysis in part Ⓔ.

PARAMETER	ESTIMATE	T FOR H0: PARAMETER=0	PR > |T|	STD ERROR OF ESTIMATE
AB VS CONTROL @ NO	-0.20333333	-1.53	0.1416	0.13288257
A VS B @ NO ALCOHOL	-0.00666667	-0.04	0.9658	0.15343958
AB VS CONTROL @ YES	-0.60500000	-4.55	0.0002	0.13288257
A VS B @ YES ALCOHOL	-0.61000000	-3.98	0.0007	0.15343958

Simple effects as in Ⓕ.

PLOT OF R*P   SYMBOL USED IS *



I ANALYSIS OF VARIANCE PROCEDURE

MEANS

ALCOHOL	N	Y
NO	18	2.83055556
YES	18	3.64000000

SUBJECT	ALCOHOL	N	Y
7	NO	3	2.67000000
8	NO	3	2.69333333
9	NO	3	3.65000000
10	NO	3	2.94333333
11	NO	3	2.19666667
12	NO	3	2.83000000
1	YES	3	3.62000000
2	YES	3	3.72000000
3	YES	3	4.40000000
4	YES	3	3.42666667
5	YES	3	3.33000000
6	YES	3	3.34333333

DRUG	N	Y
A	12	3.21583333
B	12	3.52416667
C	12	2.96583333

ALCOHOL	DRUG	N	Y
NO	A	6	2.89500000
NO	B	6	2.90166667
NO	C	6	2.69500000
YES	A	6	3.53666667
YES	B	6	4.14666667
YES	C	6	3.23666667

Note: Only the MEANS output is included from the PROC ANOVA output. Although all the appropriate SS have been obtained via PROC GLM, PROC ANOVA would be less costly.

UREA SYNTHESIS DATA - Analysis of a repeated measures experiment
(Brogan & Kutner, 1980. The American Statistician 34:229-232)

```
DATA SHUNT;
INPUT PRE POST @@;
SUBJECT = _N_;
IF _N_ LE 8 THEN GROUP='NEW';
ELSE GROUP='OLD';
XO=1;
PREPOST = POST - PRE;
TMT     = POST + PRE;
TMT_X_PP= PREPOST;
IF GROUP='OLD' THEN PREPOST=-PREPOST;
CARDS;
```

These statements calculate the sums and differences required for the appropriate t-tests. See printout in part A and B.

```
PROC SORT; BY GROUP;
```

A
```
PROC PRINT; BY GROUP;
```

B
```
PROC TTEST; CLASS GROUP;
     VAR TMT PREPOST TMT_X_PP;
```

This gives the three appropriate t-tests for treatment effects, time effects and interaction.

C
```
PROC GLM; CLASS GROUP;
     MODEL  TMT PREPOST TMT_X_PP = XO GROUP / NOINT;
     LSMEANS GROUP / STDERR;
     ESTIMATE 'NEW VS STD' GROUP 1 -1;
```

Same analysis as in B except the analyses are performed using 1-way ANOVA. Compare with E. Note that more than one dependent variable may appear to the left of the equal sign in the MODEL statement.

D
```
DATA REPEAT; SET SHUNT;
Y= PRE; TIME=1; OUTPUT;
Y=POST; TIME=2; OUTPUT;
DROP PRE POST TMT PREPOST TMT_X_PP;
```

Rearrange data for a "split-plot" type analysis.

```
PROC PRINT; VAR SUBJECT XO GROUP TIME Y;
```

E
```
PROC GLM; CLASS GROUP TIME SUBJECT;
     MODEL  Y = XO GROUP SUBJECT(GROUP)
                 TIME TIME*GROUP / NOINT SS1 SS2 SS3 SS4 P;
     MEANS GROUP SUBJECT(GROUP) TIME TIME*GROUP / DEPONLY;
     TEST  H=GROUP E=SUBJECT(GROUP) / HTYPE=1 ETYPE=1;
     OUTPUT OUT=PLOT RESIDUAL=RES PREDICTED=P;
```

Combined ANOVA Table with two error terms.

F
```
PROC PLOT; PLOT RES*P='*' / VREF=0;
```

Residual plot.

Ⓐ

GROUP=NEW

OBS	PRE	POST	SUBJECT	XO	PREPOST	TMT	TMT_X_PP
1	51	48	1	1	-3	99	-3
2	35	55	2	1	20	90	20
3	66	60	3	1	-6	126	-6
4	40	35	4	1	-5	75	-5
5	39	36	5	1	-3	75	-3
6	46	43	6	1	-3	89	-3
7	52	46	7	1	-6	98	-6
8	42	54	8	1	12	96	12

GROUP=OLD

OBS	PRE	POST	SUBJECT	XO	PREPOST	TMT	TMT_X_PP
9	34	16	9	1	18	50	-18
10	40	36	10	1	4	76	-4
11	34	16	11	1	18	50	-18
12	36	18	12	1	18	54	-18
13	38	32	13	1	6	70	-6
14	32	14	14	1	18	46	-18
15	44	20	15	1	24	64	-24
16	50	43	16	1	7	93	-7
17	60	45	17	1	15	105	-15
18	63	67	18	1	-4	130	4
19	50	36	19	1	14	86	-14
20	42	34	20	1	8	76	-8
21	43	32	21	1	11	75	-11

PRE = preoperative response

POST = postoperative response

Treatment		Pre	Post
	New (n=8)	μ_{11}	μ_{12}
	Old (n=13)	μ_{21}	μ_{22}

Note that: TMT_X_PP = $y_{ij2} - y_{ij1} = d_{ij}$ (= within subject differences)

$$\text{PREPOST} = \begin{cases} d_{ij} & \text{if group = NEW} \\ -d_{ij} & \text{if group = OLD} \end{cases}$$

PREPOST is needed since PROC TTEST automatically takes differences, thus $\bar{d}_1 - (-\bar{d}_2) = \bar{d}_1 + \bar{d}_2$

TMT = $y_{ij1} + y_{ij2} = z_{ij}$ (= within subject sums)

The assumed model may be written as:

$$y_{ijk} = \mu_{ik} + \delta_{j(i)} + \epsilon_{ijk} ,$$

$$\mu_{ik} = \mu + \alpha_i + \tau_k + (\alpha\tau)_{ik}$$

$$i = 1,2; \quad j = 1,\cdots,n_i; \quad k = 1,2$$

where:

μ_{ik} = cell mean of ith treatment, kth time combination

μ = overall mean

α_i = ith treatment effect

τ_k = kth time effect

$(\alpha\tau)_{ik}$ = treatment by time interaction

$\delta_{j(i)}$ = effect of subject j nested within ith treatment group

ϵ_{ijk} = random error

$\epsilon_{ijk} \sim N(0,\sigma_\epsilon^2)$

$\delta_{j(i)} \sim N(0,\sigma_\delta^2)$

$y_{ijk} \sim N(\mu_{ik},\sigma_\delta^2+\sigma_\epsilon^2)$.

(B) TTEST PROCEDURE

VARIABLE: TMT t-test uses $\bar{z}_1 - \bar{z}_2$ "difference of sums"

GROUP	N	MEAN	STD DEV	STD ERROR	MINIMUM	MAXIMUM
NEW	8	93.50000000	16.16875293	5.71651742	75.00000000	126.0000000
OLD	13	75.00000000	24.23839929	6.72252242	46.00000000	130.0000000

VARIANCES	T	DF	PROB > |T|
UNEQUAL	2.0964	18.8	0.0498
EQUAL	1.9344	19.0	0.0721

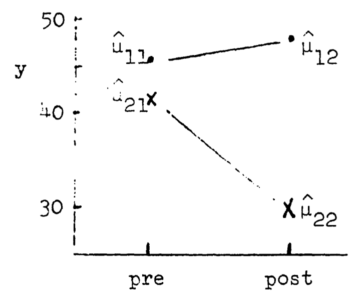
FOR H0: VARIANCES ARE EQUAL, F'= 2.25 WITH 12 AND 7 DF PROB > F'= 0.2891

VARIABLE: PREPOST t-test uses $\bar{d}_1 + \bar{d}_2$ "sum of differences"

GROUP	N	MEAN	STD DEV	STD ERROR	MINIMUM	MAXIMUM
NEW	8	0.75000000	9.73579551	3.44212351	-6.00000000	20.00000000
OLD	13	12.07692308	7.63174860	2.11666628	-4.00000000	24.00000000

VARIANCES	T	DF	PROB > |T|
UNEQUAL	-2.8031	12.3	0.0157
EQUAL	-2.9767	19.0	0.0078

FOR H0: VARIANCES ARE EQUAL, F'= 1.63 WITH 7 AND 12 DF PROB > F'= 0.4375



These are not helpful due to the significant interaction.

VARIABLE: TMT_X_PP t-test uses $\bar{d}_1 - \bar{d}_2$ "difference of differences"

GROUP	N	MEAN	STD DEV	STD ERROR	MINIMUM	MAXIMUM
NEW	8	0.75000000	9.73579551	3.44212351	-6.00000000	20.00000000
OLD	13	-12.07692308	7.63174860	2.11666628	-24.00000000	4.00000000

VARIANCES	T	DF	PROB > |T|
UNEQUAL	3.1743	12.3	0.0078
EQUAL	3.3709	19.0	0.0032

Since the interaction is clearly important we need to consider simple effects

FOR H0: VARIANCES ARE EQUAL, F'= 1.63 WITH 7 AND 12 DF PROB > F'= 0.4375

(C)

GENERAL LINEAR MODELS PROCEDURE

CLASS LEVEL INFORMATION

CLASS	LEVELS	VALUES
GROUP	2	NEW OLD

NUMBER OF OBSERVATIONS IN DATA SET = 21

An alternative approach to finding the same information as in parts (A) or (E).

DEPENDENT VARIABLE: TMT

SOURCE	DF	SUM OF SQUARES	MEAN SQUARE	F VALUE
MODEL	2	143063.00000000	71531.50000000	153.05
ERROR	19	8880.00000000	467.36842105	PR > F
UNCORRECTED TOTAL	21	151943.00000000		0.0001

R-SQUARE	C.V.	STD DEV	TMT MEAN
0.941557	26.3490	21.61870535	82.04761905

SOURCE	DF	TYPE I SS	F VALUE	PR > F
XO	1	141368.04761905	302.48	0.0001
GROUP	1	1694.95238095	3.63	0.0721

SOURCE	DF	TYPE IV SS	F VALUE	PR > F
XO	0	0.00000000	.	.
GROUP	1	1694.95238095	3.63	0.0721

PARAMETER	ESTIMATE	T FOR H0: PARAMETER=0	PR > |T|	STD ERROR OF ESTIMATE
NEW VS STD	18.50000000	1.90	0.0721	9.71454936

GENERAL LINEAR MODELS PROCEDURE

©

DEPENDENT VARIABLE: PREPOST

SOURCE	DF	SUM OF SQUARES	MEAN SQUARE	F VALUE
MODEL	2	1900.57692308	950.28846154	13.25
ERROR	19	1362.42307692	71.70647773	PR > F
UNCORRECTED TOTAL	21	3263.00000000		0.0002

R-SQUARE	C.V.	STD DEV	PREPOST MEAN
0.582463	109.0965	8.46796775	7.76190476

SOURCE	DF	TYPE I SS	F VALUE	PR > F
XO	1	1265.19047619	17.64	0.0005
GROUP	1	635.38644689	8.86	0.0078

SOURCE	DF	TYPE IV SS	F VALUE	PR > F
XO	0	0.00000000	.	.
GROUP	1	635.38644689	8.86	0.0078

PARAMETER	ESTIMATE	T FOR H0: PARAMETER=0	PR > |T|	STD ERROR OF ESTIMATE
NEW VS STD	-11.32692308	-2.98	0.0078	3.80515343

DEPENDENT VARIABLE: TMT_X_PP

SOURCE	DF	SUM OF SQUARES	MEAN SQUARE	F VALUE
MODEL	2	1900.57692308	950.28846154	13.25
ERROR	19	1362.42307692	71.70647773	PR > F
UNCORRECTED TOTAL	21	3263.00000000		0.0002

R-SQUARE	C.V.	STD DEV	TMT_X_PP MEAN
0.582463	117.7664	8.46796775	-7.19047619

SOURCE	DF	TYPE I SS	F VALUE	PR > F
XO	1	1085.76190476	15.14	0.0010
GROUP	1	814.81501832	11.36	0.0032

SOURCE	DF	TYPE IV SS	F VALUE	PR > F
XO	0	0.00000000	.	.
GROUP	1	814.81501832	11.36	0.0032

PARAMETER	ESTIMATE	T FOR H0: PARAMETER=0	PR > |T|	STD ERROR OF ESTIMATE
NEW VS STD	12.82692308	3.37	0.0032	3.80515343

LEAST SQUARES MEANS

GROUP	TMT LSMEAN	STD ERR LSMEAN	PROB > |T| H0:LSMEAN=0
NEW	93.5000000	7.6433666	0.0001
OLD	75.0000000	5.9959501	0.0001

GROUP	PREPOST LSMEAN	STD ERR LSMEAN	PROB > |T| H0:LSMEAN=0
NEW	0.7500000	2.9938787	0.8049
OLD	12.0769231	2.3485917	0.0001

GROUP	TMT_X_PP LSMEAN	STD ERR LSMEAN	PROB > |T| H0:LSMEAN=0
NEW	0.7500000	2.9938787	0.8049
OLD	-12.0769231	2.3485917	0.0001

D

OBS	SUBJECT	XO	GROUP	TIME	Y
1	1	1	NEW	1	51
2	1	1	NEW	2	46
3	2	1	NEW	1	35
4	2	1	NEW	2	55
5	3	1	NEW	1	66
6	3	1	NEW	2	60
7	4	1	NEW	1	40
8	4	1	NEW	2	35
9	5	1	NEW	1	39
10	5	1	NEW	2	36
11	6	1	NEW	1	46
12	6	1	NEW	2	43
13	7	1	NEW	1	52
14	7	1	NEW	2	46
15	8	1	NEW	1	42
16	8	1	NEW	2	54
17	9	1	OLD	1	34
18	9	1	OLD	2	16
19	10	1	OLD	1	40
20	10	1	OLD	2	36
21	11	1	OLD	1	34
22	11	1	OLD	2	16
23	12	1	OLD	1	36
24	12	1	OLD	2	18
25	13	1	OLD	1	38
26	13	1	OLD	2	32
27	14	1	OLD	1	32
28	14	1	OLD	2	14
29	15	1	OLD	1	44
30	15	1	OLD	2	20
31	16	1	OLD	1	50
32	16	1	OLD	2	43
33	17	1	OLD	1	60
34	17	1	OLD	2	45
35	18	1	OLD	1	63
36	18	1	OLD	2	67
37	19	1	OLD	1	50
38	19	1	OLD	2	36
39	20	1	OLD	1	42
40	20	1	OLD	2	34
41	21	1	OLD	1	43
42	21	1	OLD	2	32

E

GENERAL LINEAR MODELS PROCEDURE

CLASS LEVEL INFORMATION

CLASS	LEVELS	VALUES
GROUP	2	NEW OLD
TIME	2	1 2
SUBJECT	21	1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21

NUMBER OF OBSERVATIONS IN DATA SET = 42

E

GENERAL LINEAR MODELS PROCEDURE

DEPENDENT VARIABLE: Y

SOURCE	DF	SUM OF SQUARES	MEAN SQUARE	F VALUE
MODEL	23	76921.78846154	3344.42558528	93.28
ERROR	19	681.21153846	35.85323887	PR > F
UNCORRECTED TOTAL	42	77603.00000000		0.0001

R-SQUARE	C.V.	STD DEV	Y MEAN
0.991222	14.5958	5.98775742	41.02380952

SOURCE	DF	TYPE I SS	F VALUE	PR > F
XO	1	70684.02380952	1971.48	~~0.0001~~
GROUP	1	847.47619048	23.64	~~0.0001~~
SUBJECT(GROUP)	19	4440.00000000	6.52	~~0.0001~~
TIME	1	542.88095238	15.14	0.0010
GROUP*TIME	1	407.40750916	11.36	0.0032

$$t = \frac{\left(\frac{8}{21}\right)(-0.75) + \left(\frac{13}{21}\right)(12.0769)}{\left\{2(35.85324)\left[\left(\frac{8}{21}\right)^2\left(\frac{1}{8}\right)+\left(\frac{13}{21}\right)^2\left(\frac{1}{13}\right)\right]\right\}^{\frac{1}{2}}}$$

$$= 3.89 \Rightarrow F = (3.89)^2 = 15.14 .$$

SOURCE	DF	TYPE II SS	F VALUE	PR > F
XO	0	0.00000000	.	.
GROUP	1	847.47619048	23.64	~~0.0001~~
SUBJECT(GROUP)	19	4440.00000000	6.52	~~0.0001~~
TIME	1	542.88095238	15.14	0.0010
GROUP*TIME	1	407.40750916	11.36	0.0032

The Type I and II SS for TIME test equality of the weighted means -- weighted according to sample size. This is probably not desired and the Type III or IV should be used as they test equality of unweighted means.

SOURCE	DF	TYPE III SS	F VALUE	PR > F
XO	0	0.00000000	.	.
GROUP	1	847.47619048	23.64	0.0001
SUBJECT(GROUP)	19	4440.00000000	6.52	0.0001
TIME	1	317.69322344	8.86	0.0078
GROUP*TIME	1	407.40750916	11.36	0.0032

SOURCE	DF	TYPE IV SS	F VALUE	PR > F
XO	0	0.00000000	.	.
GROUP	1	847.47619048	23.64	0.0001
SUBJECT(GROUP)	19	4440.00000000	6.52	0.0001
TIME	1	317.69322344	8.86	0.0078
GROUP*TIME	1	407.40750916	11.36	0.0032

TESTS OF HYPOTHESES USING THE TYPE I MS FOR SUBJECT(GROUP) AS AN ERROR TERM

SOURCE	DF	TYPE I SS	F VALUE	PR > F
GROUP	1	847.47619048	3.03	0.0721

Compare with the results of B and C.

(E) GENERAL LINEAR MODELS PROCEDURE

MEANS

GROUP	N	Y
NEW	16	46.7500000
OLD	26	37.5000000

SUBJECT	GROUP	N	Y
1	NEW	2	49.5000000
2	NEW	2	45.0000000
3	NEW	2	63.0000000
4	NEW	2	37.5000000
5	NEW	2	37.5000000
6	NEW	2	44.5000000
7	NEW	2	49.0000000
8	NEW	2	48.0000000
9	OLD	2	25.0000000
10	OLD	2	38.0000000
11	OLD	2	25.0000000
12	OLD	2	27.0000000
13	OLD	2	35.0000000
14	OLD	2	23.0000000
15	OLD	2	32.0000000
16	OLD	2	46.5000000
17	OLD	2	52.5000000
18	OLD	2	65.0000000
19	OLD	2	43.0000000
20	OLD	2	38.0000000
21	OLD	2	37.5000000

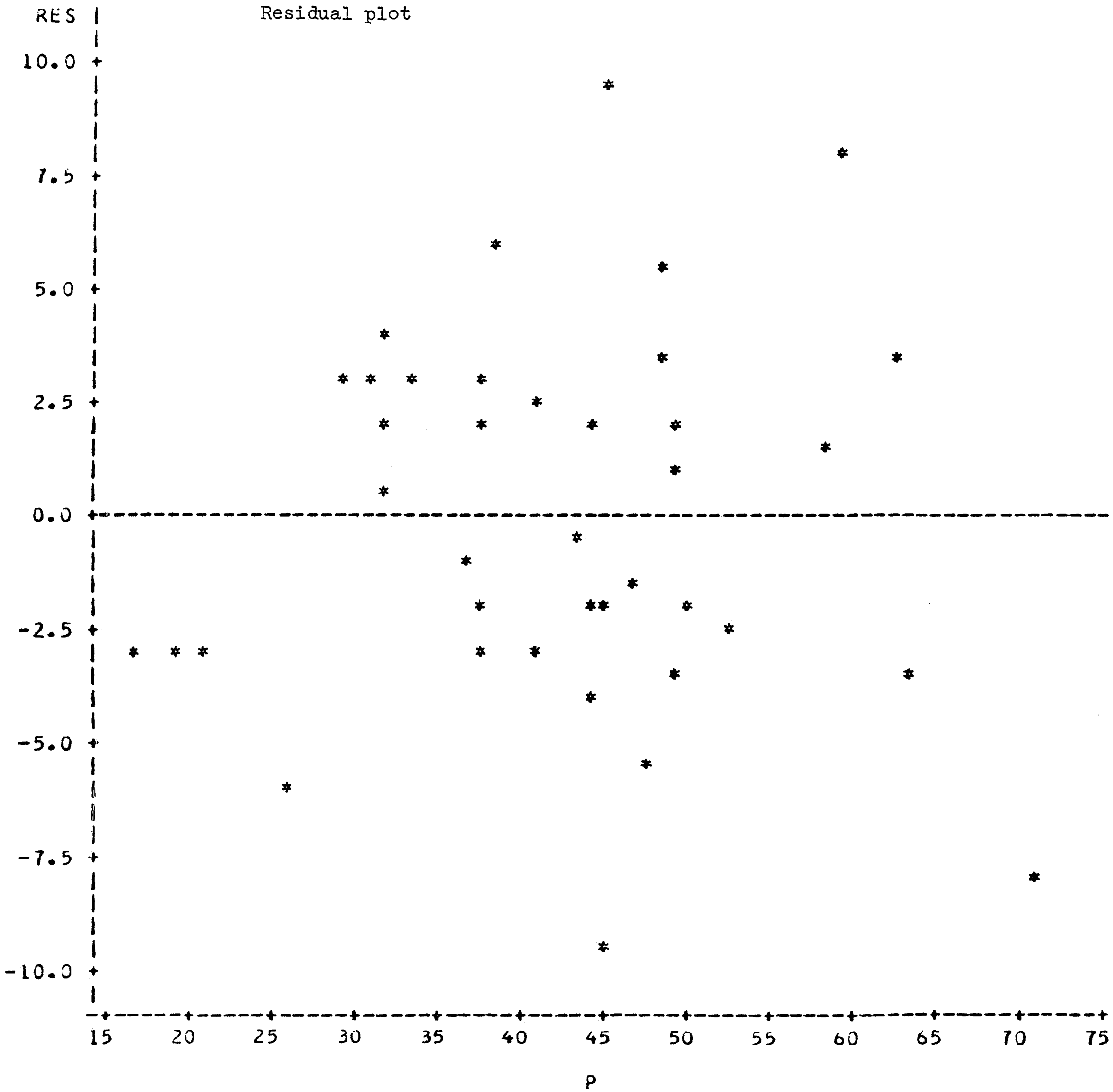
TIME	N	Y
1	21	44.6190476
2	21	37.4285714

GROUP	TIME	N	Y
NEW	1	8	46.3750000
NEW	2	8	47.1250000
OLD	1	13	43.5384615
OLD	2	13	31.4615385

These means are plotted in part (A).

(F) PLOT OF RES*P   SYMBOL USED IS *

Residual plot



NOTE: 3 OBS HIDDEN

HEMOGLOBIN DATA — Analysis of a 2-Period Cross-over Design
(Grizzle, 1965, Biometrics 21: 467-480)

```
DATA HEMO;
INPUT Y1 Y2 @@;
SUBJECT = _N_;
IF _N_ LE 6 THEN GROUP='A1_B2';
ELSE GROUP='B1_A2';
TMT   = Y2-Y1;
RESID= Y2+Y1;
TREND= TMT;
IF GROUP='B1_A2' THEN TREND=-TMT;
XO = 1;
CARDS;
```

These statements calculate the sums and differences required for the appropriate t-tests. See printout for (A) and (B).

```
PROC SORT; BY GROUP;
```

(A)
```
PROC PRINT N; BY GROUP;
```

(B)
```
PROC TTEST; CLASS GROUP;
     VAR TMT RESID TREND Y1;
```

PROC TTEST gives the appropriate four t-tests for treatment effects, carry-over effects, time trend and treatment differences within first period only.

(C)
```
PROC GLM; CLASS GROUP;
     MODEL TMT RESID TREND Y1 = XO GROUP / NOINT;
     LSMEANS GROUP / STDERR;
     ESTIMATE 'CONTRAST' GROUP 1 -1;
```

This performs the same analyses as those found in part (B). Compare also with the results in part (E).

(D)
```
DATA AOV; SET HEMO;
     Y=Y1; PERIOD=1; IF _N_ LE 6 THEN TRT='A'; ELSE TRT='B'; OUTPUT;
     Y=Y2; PERIOD=2; IF _N_ LE 6 THEN TRT='B'; ELSE TRT='A'; OUTPUT;
     DROP Y1 Y2 TMT RESID TREND;
```

Rearranging the data so that the "usual" ANOVA table may be constructed.

```
PROC PRINT; VAR SUBJECT XO GROUP PERIOD TRT Y;
```

(E)
```
PROC GLM; CLASS TRT PERIOD SUBJECT GROUP;
     MODEL  Y = XO GROUP SUBJECT(GROUP)
                PERIOD TRT / NOINT P SS1 SS2 SS3 SS4;
     MEANS GROUP SUBJECT(GROUP) PERIOD TRT / DEPONLY;
     TEST H=GROUP E=SUBJECT(GROUP) / HTYPE=1 ETYPE=1;
     ESTIMATE 'TRT' TRT 1 -1;
     ESTIMATE 'PERIOD' PERIOD 1 -1;
```

Combined ANOVA table with two error terms.

(F)
```
PROC PLOT; PLOT RES*P='*' / VREF=0;
```

Residual plot.

A

GROUP=A1_B2

OBS	Y1	Y2	SUBJECT	TMT	RESID	TREND	XO
1	0.2	1.0	1	0.8	1.2	0.8	1
2	0.0	-0.7	2	-0.7	-0.7	-0.7	1
3	-0.8	0.2	3	1.0	-0.6	1.0	1
4	0.6	1.1	4	0.5	1.7	0.5	1
5	0.3	0.4	5	0.1	0.7	0.1	1
6	1.5	1.2	6	-0.3	2.7	-0.3	1

N=6

GROUP=B1_A2

OBS	Y1	Y2	SUBJECT	TMT	RESID	TREND	XO
7	1.3	0.9	7	-0.4	2.2	0.4	1
8	-2.3	1.0	8	3.3	-1.3	-3.3	1
9	0.0	0.6	9	0.6	0.6	-0.6	1
10	-0.8	-0.3	10	0.5	-1.1	-0.5	1
11	-0.4	-1.0	11	-0.6	-1.4	0.6	1
12	-2.9	1.7	12	4.6	-1.2	-4.6	1
13	-1.9	-0.3	13	1.6	-2.2	-1.6	1
14	-2.9	0.9	14	3.8	-2.0	-3.8	1

N=8

Y1 are the responses during period 1

Y2 are the responses during period 2

Group (Sequence)

		1(n=6)	2(n=8)
Period	1	A μ_{11}	B μ_{21}
	2	B μ_{12}	A μ_{22}

TMT = $y_{ij2} - y_{ij1} = d_{ij}$ (= within subject differences)

RESID = $y_{ij2} + y_{ij1} = z_{ij}$ (= within subject sums)

$$\text{TREND} = \begin{cases} d_{ij} & \text{if sequence is AB} \\ -d_{ij} & \text{if sequence is BA} \end{cases}$$

The assumed model appears as:

$$y_{ijk} = u_{ik} + \xi_{ij} + \epsilon_{ijk}$$

where

$$u_{ik} = \mu + \pi_k + \phi_\ell + \lambda_\ell$$

$j = 1, \cdots, n_i; \quad i = 1,2; \quad k = 1,2; \quad \ell = 1,2$

and

μ = general mean,
ξ_{ij} = the effect of the j-th patient (subject) within the i-th sequence, which, for the sake of testing hypotheses, we must assume to be a normally distributed random variable with mean 0 and variance σ_s^2,
π_k = the effect of the k-th period,
ϕ_l = the direct effect of the l-th drug,
λ_l = the residual effect of the l-th drug, and
ϵ_{ijk} = the random fluctuation which is normally distributed with mean 0 and variance σ_e^2, and is independent of the ξ_{ij}.

The assumptions made about ξ_{ij} and ϵ_{ijk} imply that the variance of an observation is $\sigma_s^2 + \sigma_e^2$ and that two observations on an individual have covariance σ_s^2. Observations made on different subjects are independent.

Note:

(i) $(\bar{y}_{1\cdot 1} - \bar{y}_{1\cdot 2}) - (\bar{y}_{2\cdot 1} - \bar{y}_{2\cdot 2}) = 2(\phi_1 - \phi_2) + (\lambda_2 - \lambda_1) + (\bar{\epsilon}_{1\cdot 1} - \bar{\epsilon}_{1\cdot 2} - \bar{\epsilon}_{2\cdot 1} + \bar{\epsilon}_{2\cdot 2})$

(ii) $(\bar{y}_{1\cdot 1} - \bar{y}_{1\cdot 2}) + (\bar{y}_{2\cdot 1} - \bar{y}_{2\cdot 2}) = 2(\pi_1 - \pi_2) - (\lambda_1 + \lambda_2) + (\bar{\epsilon}_{1\cdot 1} - \bar{\epsilon}_{1\cdot 2} + \bar{\epsilon}_{2\cdot 1} - \bar{\epsilon}_{2\cdot 2})$

(iii) $(\bar{y}_{1\cdot 1} + \bar{y}_{1\cdot 2}) - (\bar{y}_{2\cdot 1} + \bar{y}_{2\cdot 2}) = (\lambda_2 - \lambda_1) + 2(\bar{\xi}_{1\cdot} - \bar{\xi}_{2\cdot}) + (\bar{\epsilon}_{1\cdot 1} + \bar{\epsilon}_{1\cdot 2} - \bar{\epsilon}_{2\cdot 1} - \bar{\epsilon}_{2\cdot 2})$

(iv) $\bar{y}_{1\cdot 1} - \bar{y}_{2\cdot 1} = (\phi_1 - \phi_2) + (\bar{\xi}_{1\cdot} - \bar{\xi}_{2\cdot}) + (\bar{\epsilon}_{1\cdot 1} - \bar{\epsilon}_{2\cdot 1})$

Now, i) through iv) may be recognized as:

(i) $\bar{d}_1 - \bar{d}_2$, a test of treatment effects if there is no carryover effect

(ii) $\bar{d}_1 + \bar{d}_2$, a test of period effects if there is no carryover effect

(iii) $\bar{z}_1 - \bar{z}_2$, tests for carryover effects

(iv) tests for treatment effects based upon the first period data only

Ⓑ TTEST PROCEDURE

VARIABLE: TMT   t-test uses $\bar{d}_1 - \bar{d}_2$ "difference of differences"

GROUP	N	MEAN	STD DEV	STD ERROR	MINIMUM	MAXIMUM
A1_B2	6	0.23333333	0.65625198	0.26791375	-0.70000000	1.00000000
B1_A2	8	1.67500000	1.99051321	0.70375270	-0.60000000	4.60000000

VARIANCES	T	DF	PROB > |T|
UNEQUAL	-1.9145	8.9	0.0882
EQUAL	-1.6915	12.0	0.1165

FOR HO: VARIANCES ARE EQUAL, F'= 9.20 WITH 7 AND 5 DF   PROB > F'= 0.0265

VARIABLE: RESID   t-test uses $\bar{z}_1 - \bar{z}_2$ "difference of sums"

GROUP	N	MEAN	STD DEV	STD ERROR	MINIMUM	MAXIMUM
A1_B2	6	0.83333333	1.32614730	0.54139737	-0.70000000	2.70000000
B1_A2	8	-0.80000000	1.47454593	0.52133071	-2.20000000	2.20000000

VARIANCES	T	DF	PROB > |T|
UNEQUAL	2.1732	11.5	0.0515
EQUAL	2.1379	12.0	0.0538

There appear to be important carry-over effects. Thus, the above test for treatment effects is not free of residual effects and we must resort to the results of the first period only (variable Y1 below).

FOR HO: VARIANCES ARE EQUAL, F'= 1.24 WITH 7 AND 5 DF   PROB > F'= 0.8437

VARIABLE: TREND   t-test uses $\bar{d}_1 + \bar{d}_2$ "sum of differences"

GROUP	N	MEAN	STD DEV	STD ERROR	MINIMUM	MAXIMUM
A1_B2	6	0.23333333	0.65625198	0.26791375	-0.70000000	1.00000000
B1_A2	8	-1.67500000	1.99051321	0.70375270	-4.60000000	0.60000000

VARIANCES	T	DF	PROB > |T|
UNEQUAL	2.5342	8.9	0.0323
EQUAL	2.2390	12.0	0.0449

FOR HO: VARIANCES ARE EQUAL, F'= 9.20 WITH 7 AND 5 DF   PROB > F'= 0.0265

VARIABLE: Y1   t-test uses $\bar{y}_{1.1} - \bar{y}_{2.1}$ "independent two-sample t-test for the first period only"

GROUP	N	MEAN	STD DEV	STD ERROR	MINIMUM	MAXIMUM
A1_B2	6	0.30000000	0.75365775	0.30767949	-0.80000000	1.50000000
B1_A2	8	-1.23750000	1.50990775	0.53383301	-2.90000000	1.30000000

VARIANCES	T	DF	PROB > |T|
UNEQUAL	2.4953	10.8	0.0302
EQUAL	2.2746	12.0	0.0421

There appear to be treatment effects present. Note: Had we chosen to ignore residual effects the conclusion would likely have been different.

FOR HO: VARIANCES ARE EQUAL, F'= 4.01 WITH 7 AND 5 DF   PROB > F'= 0.1453

GENERAL LINEAR MODELS PROCEDURE

DEPENDENT VARIABLE: TMT

SOURCE	DF	SUM OF SQUARES	MEAN SQUARE	F VALUE
MODEL	2	22.77166667	11.38583333	4.57
ERROR	12	29.88833333	2.49069444	PR > F
UNCORRECTED TOTAL	14	52.66000000		0.0334

R-SQUARE	C.V.	STD DEV	TMT MEAN
0.432428	149.2886	1.57819341	1.05714286

SOURCE	DF	TYPE I SS	F VALUE	PR > F
XO	1	15.64571429	6.28	0.0276
GROUP	1	7.12595238	2.86	0.1165

PARAMETER	ESTIMATE	T FOR HO: PARAMETER=0	PR > |T|	STD ERROR OF ESTIMATE
CONTRAST	-1.44166667	-1.69	0.1165	0.85232186

DEPENDENT VARIABLE: RESID

SOURCE	DF	SUM OF SQUARES	MEAN SQUARE	F VALUE
MODEL	2	9.28666667	4.64333333	2.32
ERROR	12	24.01333333	2.00111111	PR > F
UNCORRECTED TOTAL	14	33.30000000		0.1406

R-SQUARE	C.V.	STD DEV	RESID MEAN
0.278879	1414.6063	1.41460634	-0.10000000

SOURCE	DF	TYPE I SS	F VALUE	PR > F
XO	1	0.14000000	0.07	0.7959
GROUP	1	9.14666667	4.57	0.0538

PARAMETER	ESTIMATE	T FOR HO: PARAMETER=0	PR > |T|	STD ERROR OF ESTIMATE
CONTRAST	1.63333333	2.14	0.0538	0.76397474

LEAST SQUARES MEANS

GROUP	TMT LSMEAN	STD ERR LSMEAN	PROB > |T| HO:LSMEAN=0
A1_B2	0.23333333	0.64429476	0.7235
B1_A2	1.67500000	0.55797563	0.0110

GROUP	RESID LSMEAN	STD ERR LSMEAN	PROB > |T| HO:LSMEAN=0
A1_B2	0.83333333	0.57751062	0.1746
B1_A2	-0.80000000	0.50013887	0.1357

GROUP	TREND LSMEAN	STD ERR LSMEAN	PROB > |T| HO:LSMEAN=0
A1_B2	0.23333333	0.64429476	0.7235
B1_A2	-1.67500000	0.55797563	0.0110

GROUP	Y1 LSMEAN	STD ERR LSMEAN	PROB > |T| HO:LSMEAN=0
A1_B2	0.30000000	0.51097334	0.5680
B1_A2	-1.23750000	0.44251589	0.0161

C

DEPENDENT VARIABLE: TREND

SOURCE	DF	SUM OF SQUARES	MEAN SQUARE	F VALUE
MODEL	2	22.77166667	11.38583333	4.57
ERROR	12	29.88833333	2.49069444	PR > F
UNCORRECTED TOTAL	14	52.66000000		0.0334

R-SQUARE	C.V.	STD DEV	TREND MEAN
0.432428	184.1226	1.57819341	-0.85714286

SOURCE	DF	TYPE I SS	F VALUE	PR > F
XO	1	10.28571429	4.13	0.0649
GROUP	1	12.48595238	5.01	0.0449

PARAMETER	ESTIMATE	T FOR HO: PARAMETER=0	PR > |T|	STD ERROR OF ESTIMATE
CONTRAST	1.90833333	2.24	0.0449	0.85232186

DEPENDENT VARIABLE: Y1

SOURCE	DF	SUM OF SQUARES	MEAN SQUARE	F VALUE
MODEL	2	12.79125000	6.39562500	4.08
ERROR	12	18.79875000	1.56656250	PR > F
UNCORRECTED TOTAL	14	31.59000000		0.0444

R-SQUARE	C.V.	STD DEV	Y1 MEAN
0.404915	216.3301	1.25162395	-0.57857143

SOURCE	DF	TYPE I SS	F VALUE	PR > F
XO	1	4.68642857	2.99	0.1093
GROUP	1	8.10482143	5.17	0.0421

PARAMETER	ESTIMATE	T FOR HO: PARAMETER=0	PR > |T|	STD ERROR OF ESTIMATE
CONTRAST	1.53750000	2.27	0.0421	0.67595419

D

OBS	SUBJECT	XO	GROUP	PERIOD	TRT	Y
1	1	1	A1_B2	1	A	0.2
2	1	1	A1_B2	2	B	1.0
3	2	1	A1_B2	1	A	0.0
4	2	1	A1_B2	2	B	-0.7
5	3	1	A1_B2	1	A	-0.8
6	3	1	A1_B2	2	B	0.2
7	4	1	A1_B2	1	A	0.6
8	4	1	A1_B2	2	B	1.1
9	5	1	A1_B2	1	A	0.3
10	5	1	A1_B2	2	B	0.4
11	6	1	A1_B2	1	A	1.5
12	6	1	A1_B2	2	B	1.2
13	7	1	B1_A2	1	B	1.3
14	7	1	B1_A2	2	A	0.9
15	8	1	B1_A2	1	B	-2.3
16	8	1	B1_A2	2	A	1.0
17	9	1	B1_A2	1	B	0.0
18	9	1	B1_A2	2	A	0.6
19	10	1	B1_A2	1	B	-0.8
20	10	1	B1_A2	2	A	-0.3
21	11	1	B1_A2	1	B	-0.4
22	11	1	B1_A2	2	A	-1.0
23	12	1	B1_A2	1	B	-2.9
24	12	1	B1_A2	2	A	1.7
25	13	1	B1_A2	1	B	-1.9
26	13	1	B1_A2	2	A	-0.3
27	14	1	B1_A2	1	B	-2.9
28	14	1	B1_A2	2	A	0.9

E

GENERAL LINEAR MODELS PROCEDURE

CLASS LEVEL INFORMATION

CLASS	LEVELS	VALUES
TRT	2	A B
PERIOD	2	1 2
SUBJECT	14	1 2 3 4 5 6 7 8 9 10 11 12 13 14
GROUP	2	A1_B2 B1_A2

NUMBER OF OBSERVATIONS IN DATA SET = 28

DEPENDENT VARIABLE: Y

SOURCE	DF	SUM OF SQUARES	MEAN SQUARE	F VALUE
MODEL	16	28.03583333	1.75223958	1.41
ERROR	12	14.94416667	1.24534722	PR > F
UNCORRECTED TOTAL	28	42.98000000		0.2780

R-SQUARE	C.V.	STD DEV	Y MEAN
0.652300	2231.9025	1.11595126	-0.05000000

SOURCE	DF	TYPE I SS	F VALUE	PR > F
XO	1	0.07000000	0.06	~~0.8166~~
GROUP	1	4.57333333	3.67	~~0.0794~~
SUBJECT(GROUP)	12	12.00666667	0.80	~~0.6446~~
PERIOD	1	7.82285714	6.28	0.0276
TRT	1	3.56297619	2.86	0.1165

The Type I SS test equality of weighted means for the PERIOD effect. This is probably not of interest.

SOURCE	DF	TYPE II SS	F VALUE	PR > F
XO	0	0.00000000	.	.
GROUP	1	4.57333333	3.67	~~0.0794~~
SUBJECT(GROUP)	12	12.00666667	0.80	~~0.6446~~
PERIOD	1	6.24297619	5.01	0.0449
TRT	1	3.56297619	2.86	0.1165

SOURCE	DF	TYPE III SS	F VALUE	PR > F
XO	0	0.00000000	.	.
GROUP	1	4.57333333	3.67	~~0.0794~~
SUBJECT(GROUP)	12	12.00666667	0.80	~~0.6446~~
PERIOD	1	6.24297619	5.01	0.0449
TRT	1	3.56297619	2.86	0.1165

The Type III SS give the appropriate tests of unweighted means for the TRT and PERIOD effects. (See A, B and ESTIMATE on next page.)

(E)

TESTS OF HYPOTHESES USING THE TYPE I MS FOR SUBJECT(GROUP) AS AN ERROR TERM

SOURCE	DF	TYPE I SS	F VALUE	PR > F
GROUP (Residual)	1	4.57333333	4.57	0.0538

PARAMETER	ESTIMATE	T FOR H0: PARAMETER=0	PR > |T|	STD ERROR OF ESTIMATE
TRT	0.72083333	1.69	0.1165	0.42616093
PERIOD	-0.95416667	-2.24	0.0449	0.42616093

Compare with parts (B) and (C).

MEANS

GROUP	N	Y
A1_B2	12	0.41666667
B1_A2	16	-0.40000000

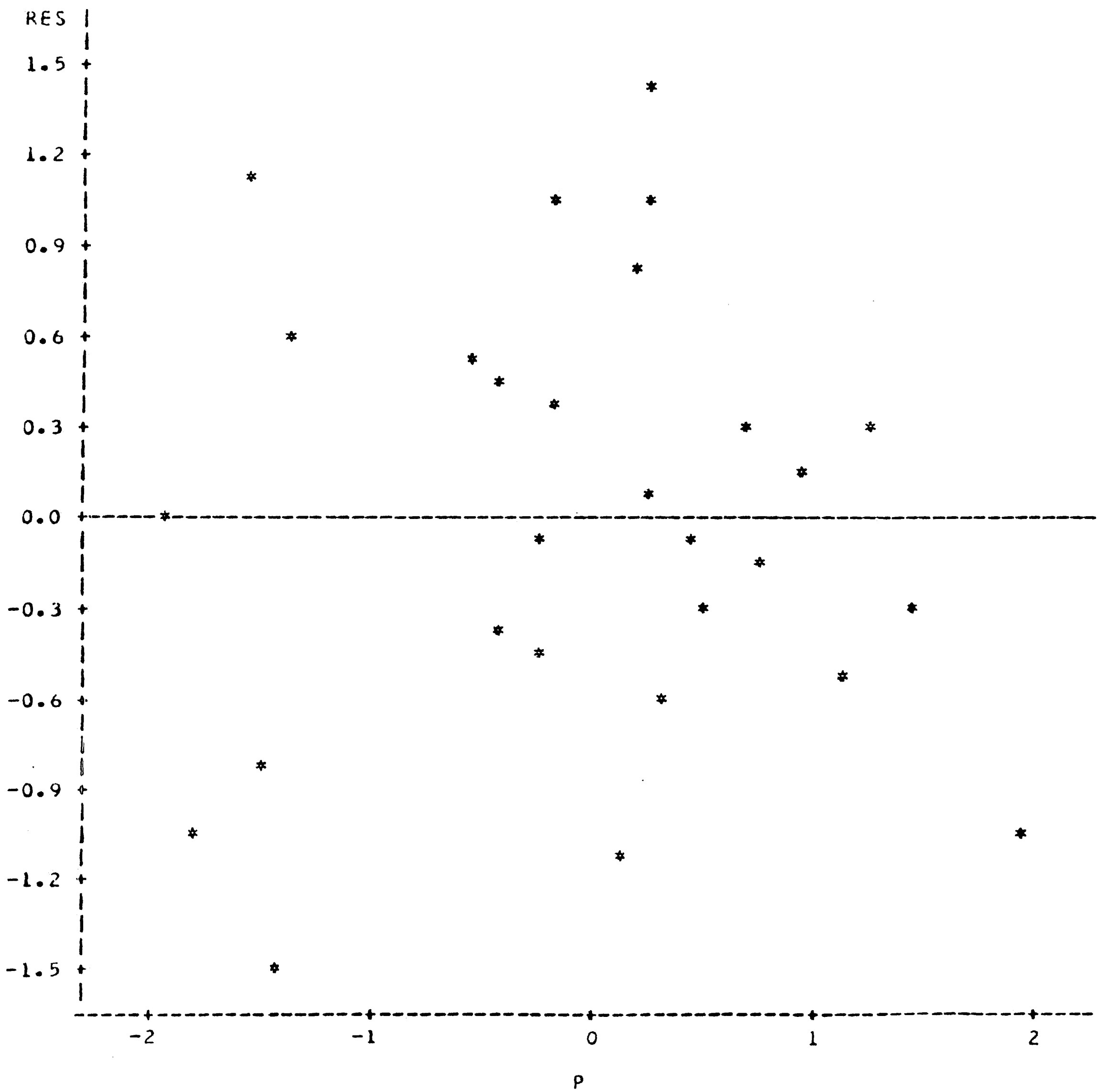
SUBJECT	GROUP	N	Y
1	A1_B2	2	0.60000000
2	A1_B2	2	-0.35000000
3	A1_B2	2	-0.30000000
4	A1_B2	2	0.85000000
5	A1_B2	2	0.35000000
6	A1_B2	2	1.35000000
7	B1_A2	2	1.10000000
8	B1_A2	2	-0.65000000
9	B1_A2	2	0.30000000
10	B1_A2	2	-0.55000000
11	B1_A2	2	-0.70000000
12	B1_A2	2	-0.60000000
13	B1_A2	2	-1.10000000
14	B1_A2	2	-1.00000000

PERIOD	N	Y
1	14	-0.57857143
2	14	0.47857143

TRT	N	Y
A	14	0.37857143
B	14	-0.47857143

(F)

PLOT OF RES*P   SYMBOL USED IS *



MILK YIELD DATA - Analysis of a 3 Period - 3 Treatment Cross-over Design Balanced for Residual Effects. (p. 134ff in Cochran and Cox, 1957)

```
DATA COW;
INPUT SQUARE COW PERIOD TRT $ YIELD RA RB RC RES;
CARDS;
```

(A)

```
PROC PRINT;
VAR SQUARE COW PERIOD TRT RA RB RC RES YIELD;
TITLE A CROSSOVER DESIGN WHEN THERE ARE POSSIBLE RESIDUAL EFFECTS;
```

(B)

```
PROC GLM;
     CLASSES SQUARE COW PERIOD TRT;
     MODEL YIELD = COW PERIOD(SQUARE) RA RB RC TRT / SS1;
     MEANS  TRT / DEPONLY;
     LSMEANS TRT / STDERR;
```

(C)

```
PROC GLM;
     CLASSES SQUARE COW PERIOD TRT;
     MODEL YIELD = COW PERIOD(SQUARE) TRT RA RB RC / SS1;
     ESTIMATE 'TRT A VS AVG(B+C)' TRT 1 -0.5 -0.5;
     ESTIMATE 'TRT B VS TRT C' TRT 0 1 -1;
     LSMEANS TRT / STDERR;
     MEANS TRT / DEPONLY;
     OUTPUT OUT=NEW PREDICTED=P RESIDUAL=R;
```

(D)

```
PROC PLOT;
     PLOT R*P='*' / VREF=0;
```

A check on the residuals reveals no obvious violations of the assumed model.

SQUARE, COW, PERIOD, TRT and RES are CLASS variables. RA, RB and RC are 0,1 indicator variables which indicate the residual effects of each of the three treatments. YIELD is the response variable, coded milk yield.

(A) CLASS, indicator and response variables

OBS	SQUARE	COW	PERIOD	TRT	RA	RB	RC	RES	YIELD
1	1	1	1	A	0	0	0	0	38
2	1	1	2	B	1	0	0	1	25
3	1	1	3	C	0	1	0	2	15
4	1	2	1	B	0	0	0	0	109
5	1	2	2	C	0	1	0	2	86
6	1	2	3	A	0	0	1	3	39
7	1	3	1	C	0	0	0	0	124
8	1	3	2	A	0	0	1	3	72
9	1	3	3	B	1	0	0	1	27
10	2	4	1	A	0	0	0	0	86
11	2	4	2	C	1	0	0	1	76
12	2	4	3	B	0	0	1	3	46
13	2	5	1	B	0	0	0	0	75
14	2	5	2	A	0	1	0	2	35
15	2	5	3	C	1	0	0	1	34
16	2	6	1	C	0	0	0	0	101
17	2	6	2	B	0	0	1	3	63
18	2	6	3	A	0	1	0	2	1

3 The model fitting residual effects before treatment effects.

A CROSSOVER DESIGN WHEN THERE ARE POSSIBLE RESIDUAL EFFECTS

GENERAL LINEAR MODELS PROCEDURE

CLASS LEVEL INFORMATION

CLASS	LEVELS	VALUES
SQUARE	2	1 2
COW	6	1 2 3 4 5 6
PERIOD	3	1 2 3
TRT	3	A B C

TRT A = Roughage diet
B = Limited grain diet
C = Full grain diet

NUMBER OF OBSERVATIONS IN DATA SET = 18

DEPENDENT VARIABLE: YIELD

SOURCE	DF	SUM OF SQUARES	MEAN SQUARE	F VALUE
MODEL	13	20163.19444444	1551.01495726	31.14
ERROR	4	199.25000000	49.81250000	PR > F
CORRECTED TOTAL	17	20362.44444444		0.0023

R-SQUARE	C.V.	STD DEV	YIELD MEAN
0.990215	12.0761	7.05779711	58.44444444

SOURCE	DF	TYPE I SS	F VALUE	PR > F
COW	5	5781.11111111	23.21	0.0047
PERIOD(SQUARE)	4	11489.11111111	57.66	0.0009
RA	1	11.75555556	0.24	0.6525
RB	1	26.66666667	0.54	0.5049
RC	0	0.00000000	.	.
TRT	2	2854.55000000	28.65	0.0043

The SS due to residual effects unadjusted for treatments is found as:

SS Residual effects (unadj.) = RA + RB + RC
= 11.7556 + 26.6667 + 0
= 38.4223 with 2 df.

Ⓒ The model-fitting treatments before residual effects.

A CROSSOVER DESIGN WHEN THERE ARE POSSIBLE RESIDUAL EFFECTS

GENERAL LINEAR MODELS PROCEDURE

DEPENDENT VARIABLE: YIELD

SOURCE	DF	SUM OF SQUARES	MEAN SQUARE	F VALUE
MODEL	13	20163.19444444	1551.01495726	31.14
ERROR	4	199.25000000	49.81250000	PR > F
CORRECTED TOTAL	17	20362.44444444		0.0023

R-SQUARE	C.V.	STD DEV	YIELD MEAN
0.990215	12.0761	7.05779711	58.44444444

SOURCE	DF	TYPE I SS	F VALUE	PR > F
COW	5	5781.11111111	23.21	0.0047
PERIOD(SQUARE)	4	11489.11111111	57.66	0.0009
TRT	2	2276.77777778	22.85	0.0065
RA	1	258.67361111	5.19	0.0849
RB	1	357.52083333	7.18	0.0553
RC	0	0.00000000	.	.

SS Residual effects (adj.) = RA + RB + RC
= 258.6736 + 357.5208 + 0
= 616.1944 with 2 df.

PARAMETER	ESTIMATE	T FOR H0: PARAMETER=0	PR > |T|	STD ERROR OF ESTIMATE
TRT A VS AVG(B+C)	-23.93750000	-6.07	0.0037	3.94542853
TRT B VS TRT C	-20.62500000	-4.53	0.0106	4.55578844

These are contrasts among the unadjusted treatment means.

Combining the results of Ⓑ and Ⓒ gives the ANOVA table found on page 135 of Cochran and Cox:

Source	df	SS	MS
Cows	5	5781.1111	
Periods w/i squares	4	11489.1111	
Treatments (unadj.)	2	2276.7778	1138.3889
Residual (adj.)	2	616.1944	308.0972
Residual (unadj.)	2	38.4223	19.1115
Treatments (adj.)	2	2854.5500	1427.2750
Error	4	199.2500	49.8125
Corrected Total	17	20362.4444	

A CROSSOVER DESIGN WHEN THERE ARE POSSIBLE RESIDUAL EFFECTS

GENERAL LINEAR MODELS PROCEDURE

MEANS = The unadjusted treatment means

TRT	N	YIELD
A	6	45.1666667
B	6	57.5000000
C	6	72.6666667

A CROSSOVER DESIGN WHEN THERE ARE POSSIBLE RESIDUAL EFFECTS

GENERAL LINEAR MODELS PROCEDURE

LEAST SQUARES MEANS = The treatment means adjusted for residual effects

TRT	YIELD LSMEAN	STD ERR LSMEAN	PROB > |T| HO:LSMEAN=0
A	42.4861111	3.1121960	0.0002
B	56.1111111	3.1121960	0.0001
C	76.7361111	3.1121960	0.0001

Notice that the adjustments are intuitively pleasing.

References

Allen, David M. and Cady, Foster B. (1982). Analyzing Experimental Data by Regression. Lifetime Learning Publications, Belmont, CA.

Aref, Suzanne and Piegorsch, Walter (1980). Analyzing Experimental Data by Regression X SAS: Annotated outputs. Technical paper BU-705-M in the Biometrics Unit series, Cornell University, Ithaca, NY.

Arneson, Valerie, Firey, Patricia A. and McCulloch, Charles E. (1978). Statistical methods X SAS. Technical paper BU-664-M in the Biometrics Unit series, Cornell University, Ithaca, NY.

Brogan, Donna R. and Kutner, Michael H. (1980). Comparative analyses of pretest-posttest research designs. The American Statistician 34: 229-232.

Cochran, William G. and Cox, Gertrude M. (1957). Experimental Designs, 2nd ed. John Wiley, New York.

Grizzle, James E. (1965). The two-period change-over design and its use in clinical trials. Biometrics 21: 467-480.

Meredith, Michael P. (1983). Annotated SAS Output (ASO). Technical paper BU-814-M in the Biometrics Unit series, Cornell University, Ithaca, NY.

SAS Institute, Inc. (1982). SAS User's Guide: Statistics. SAS Institute, Inc., Cary, NC, 584 pp.

Snedecor, George W. and Cochran, William G. (1980). Statistical Methods, 7th ed. Iowa State University Press, Ames, Iowa.